Prop.

$$\begin{array}{ccc} \tilde{X} & \ni & \tilde{x}_0 \\ p\downarrow & & \downarrow \\ X & \ni & x_0 \end{array}$$ path-connected, path-connected, loc. —"—

$H = p_* \pi_1(\tilde{X}, \tilde{x}_0) \subseteq \pi_1(X, x_0)$

1) $\tilde{X} \xrightarrow{p} X$ is normal $\iff$ $H \subseteq \pi_1(X, x_0)$ normal subgp.

2) $G(\tilde{X}) \approx N(H)/H$ $\quad N(H)$ normalizer of H in $\pi_1(X, x_0)$

$\Rightarrow G(\tilde{X}) \approx \pi_1(X, x_0)/H$ if $\tilde{X}$ normal

and: $G(\tilde{X}) \approx \pi_1(X)$ for the universal cover

Pf. $H \subseteq G$ $\quad N(H) =$ biggest subgp of G s.t. $H \trianglelefteq N(H)$ normal.

$\forall g \in N(H)$; $gHg^{-1} = H$ $\;$ & $N(H) = \{g \in G \mid gHg^{-1} = H\}$

1) Changing base pt : $\tilde{x}_0 \rightsquigarrow \tilde{x}_1 \quad \in p^{-1}(x_0)$

corresponds to $\quad [\gamma]\, p_* \pi_1(\tilde{X}, \tilde{x}_0) [\gamma]^{-1} = p_* \pi_1(\tilde{X}, \tilde{x}_1)$

where $[\gamma] \in \pi_1(X, x_0)$ lifts to a path $\tilde{x}_1 \xrightarrow{\tilde{\gamma}} \tilde{x}_0$

$\Rightarrow [\gamma] \in N(H)$ iff $p_* \pi_1(\tilde{X}, \tilde{x}_0) = p_* \pi_1(\tilde{X}, \tilde{x}_1)$

$\Updownarrow$

$\exists$ Deck transformation which takes $\tilde{x}_0$ to $\tilde{x}_1$.

$\Rightarrow$ $\tilde{X} \to X$ normal $\iff N(H) = \pi_1(X, x_0)$

(H is normal subgp of $\pi_1(X, x_0)$)

2) Define $\varphi : N(H) \to G(\tilde{X})$

$[\gamma] \longmapsto \tau_\gamma$ given by $\tilde{\gamma}(0) \longmapsto \tilde{\gamma}(1)$

homomorphism $\quad \tilde{\gamma}(0) \longmapsto \tilde{\gamma}(1) \overset{\shortparallel}{=} \tilde{\gamma}'(0) \longmapsto \tilde{\gamma}'(1)$

φ is surjective !

Kernel : $[\gamma]$ lifts to loop i.e. $p_* \pi_1(\tilde{X}, \tilde{x}_0) = H$

Notice (Prop 1.40)

G acts on Y. s.t. $\forall y \in Y \ \exists \underset{\text{nghd}}{y \in U}$ s.t. $\forall \underset{g \neq e}{g \in G} \ U \cap g(U) = \emptyset$.

(Deck transformations have this property)

1) $\begin{array}{ccc} Y & \ni & y \\ \downarrow & & \downarrow \\ Y/G & \ni & Gy \end{array}$ normal covering space

points are orbits

2) $G = \underset{\text{Deck trans.}}{G(Y)}$ if Y is path-connected

3) $G \approx \pi_1(Y/G) / p_* \pi_1(Y)$ if Y is path-connected and locally –"–

Choose base pt $y_0 \in Y$. Then a path from y_0 to $g(y_0)$ will project to a loop in Y/G.

Two paths, as above, are homotopic (Y is path-connected)

$\rightsquigarrow$ well-defined element in $\pi_1(Y/G)$.