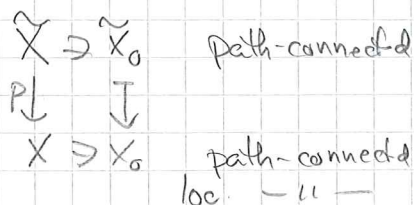


Prop



$$H = p_* \pi_1(\tilde{X}, \tilde{x}_0) \subseteq \pi_1(X, x_0)$$

$$1) \begin{array}{c} \tilde{X} \\ \downarrow p \\ X \end{array} \text{ is normal} \iff H \subseteq \pi_1(X, x_0) \text{ normal subgroup.}$$

$$2) G(\tilde{X}) \cong N(H)/H \quad N(H) \text{ normalizer of } H \text{ in } \pi_1(X, x_0)$$

$$\Rightarrow G(\tilde{X}) \cong \pi_1(X, x_0)/H \text{ if } \tilde{X} \text{ normal}$$

and: $G(\tilde{X}) \cong \pi_1(X)$ for the universal cover

Pr.

$$H \subseteq G \quad N(H) = \text{biggest subgroup of } G \text{ s.t. } H \subseteq N(H)$$

$$\forall g \in N(H); gHg^{-1} = H \quad ; \quad N(H) = \{g \in G \mid gHg^{-1} = H\} \text{ normal.}$$

$$1) \text{ Changing base pt: } \tilde{x}_0 \rightsquigarrow \tilde{x}_1 \in p^{-1}(x_0)$$

$$\text{corresponds to } [\gamma] p_* \pi_1(\tilde{X}, \tilde{x}_0) [\gamma]^{-1} = p_* \pi_1(\tilde{X}, \tilde{x}_1)$$

where $[\gamma] \in \pi_1(X, x_0)$ lifts to a path $\tilde{\gamma}_1 \rightsquigarrow \tilde{\gamma}_0$

$$\Rightarrow [\gamma] \in N(H) \text{ iff } \underline{p_* \pi_1(\tilde{X}, \tilde{x}_0)} = p_* \pi_1(\tilde{X}, \tilde{x}_1)$$



\exists Deck transformation which takes \tilde{x}_0 to \tilde{x}_1 .

$$\Rightarrow \begin{array}{c} \tilde{X} \\ \downarrow p \\ X \end{array} \text{ normal} \iff N(H) = \pi_1(X, x_0) \quad (H \text{ is normal subgroup of } \pi_1(X, x_0))$$

$$2) \text{ Define } \varphi: N(H) \rightarrow G(\tilde{X})$$

$$[\gamma] \mapsto \gamma_\gamma \text{ given by } \tilde{\gamma}(0) \mapsto \tilde{\gamma}(1)$$

$$\text{homomorphism } \tilde{\gamma}(0) \mapsto \tilde{\gamma}(1)$$

$$\tilde{\gamma}'(0) \mapsto \tilde{\gamma}'(1)$$

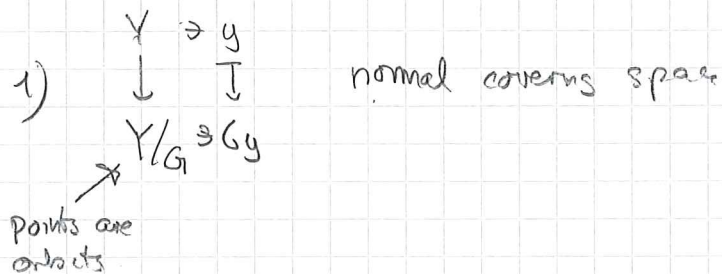
φ is surjective!

Kernel: $[\gamma]$ lifts to loop i.e. $p_* \pi_1(\tilde{X}, \tilde{x}_0) = H$

Notice (Prop 1.40)

G acts on Y s.t. $\forall y \in Y \exists g \in G$ s.t. $\forall g \in G \cup \{e\} \cup g^{-1}Gg$ $\cap g^{-1}Gg = \emptyset$

(Deck transformations have this property)



2) $G = G(Y)$ if Y is path-connected
Deck trans.

3) $G \cong \pi_1(Y/G) / P_* \pi_1(Y)$ if Y is path-connected and locally -u-

Choose base pt $y_0 \in Y$. Then a path from y_0 to $g(y_0)$ will project to a loop in Y/G .

Two paths, as above, are homotopic (Y is path-connected)
 \leadsto well-defined element in $\pi_1(Y/G)$.