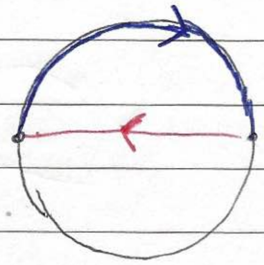
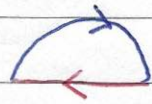
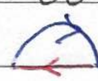
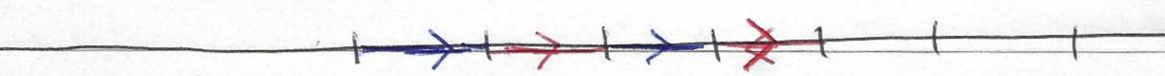
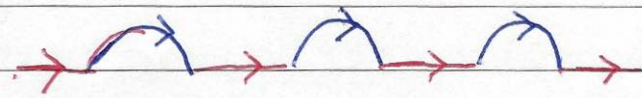
1.3.4

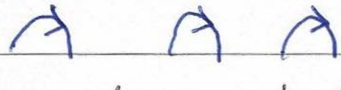
Let X be the union of a sphere and a diameter

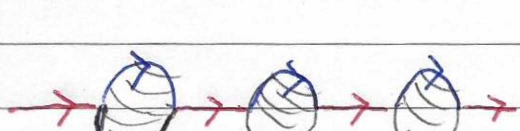


Let  be the union of a half-circle and the diameter. This is homeomorphic with S^1 and it has a simply connected covering space $\mathbb{R}$ which is mapped on  the following way



or 

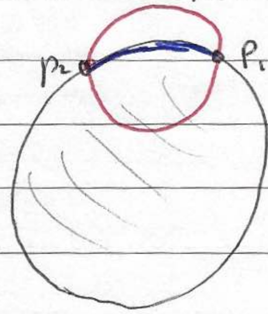
Now on each segment 
we attach a two sphere and map to the standard two sphere, so we get the following covering space Y of X



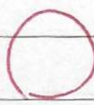
To see that this covering space is simply-connected it is homotopy equivalent with wedge of spheres
By van Kampens theorem, there is

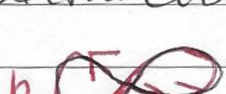
a surjective map $* \pi_1(S^2) \xrightarrow{\Phi} \pi_1(Y)$
and since S^2 is simply connected
$* \pi_1(S^2) = \{0\}$ hence $\pi_1(Y) = \{0\}$
(also Y is path-connected)

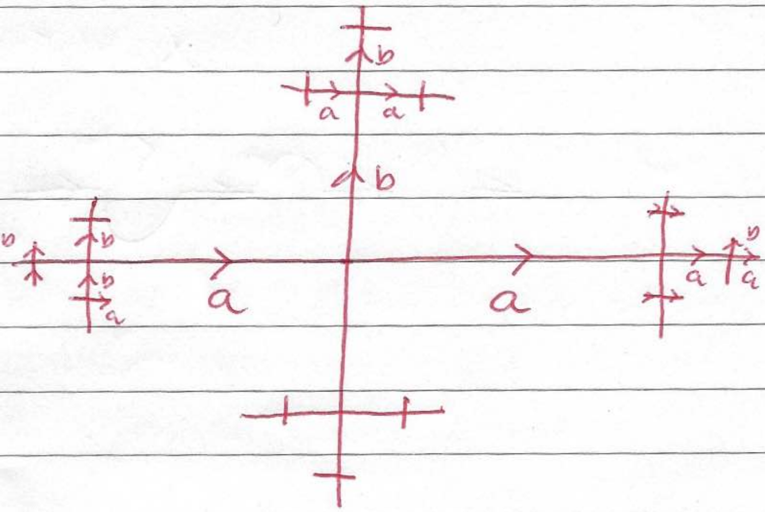
Next, let X be the union of a sphere S^2 and a circle intersecting S^2 in two points



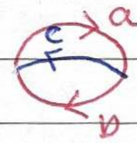
X

Let  be the circle and  an arc on S^2 between the two points p_1 and p_2.

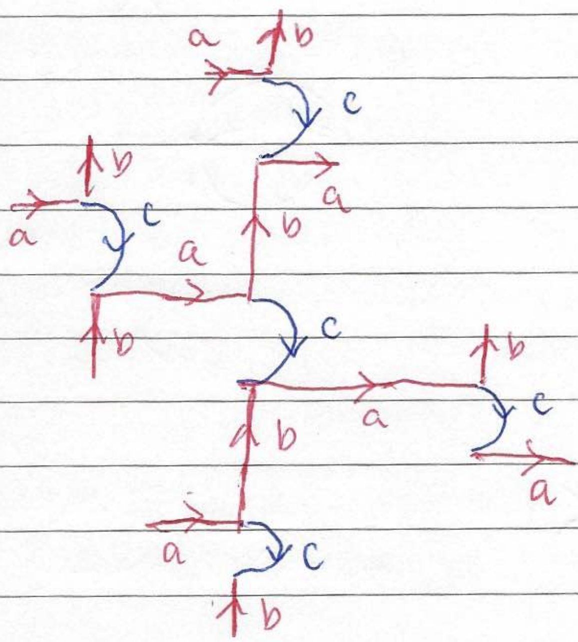
In the text book there is a construction of a simply-connected covering space of the wedge of two circles 



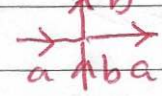
This covering space X is the union of ~~covering~~ spaces $X = \bigcup_n X_n$ with $X_n \subset X_{n+1}$

We can modify X such that we get a coveringspace Y of 

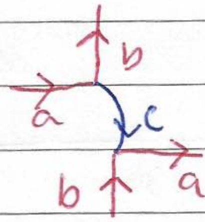
Put Y_n equal ...

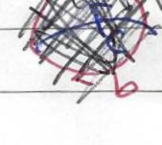


That is we modify X_n by at each vertex



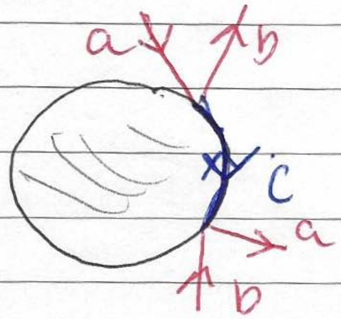
inserting an arc c



~~Now each Y_n is a covering space of~~ 

~~And so is~~ Let $Y = \bigcup_n Y_n$. Y becomes a covering space of

Finally we attach ~~∞~~ 2-spheres such that the arcs c becomes arcs in these spheres



So we get for each n a space Z_n of X and $Z = \bigcup_n Z_n$, ($Z_n \subset Z_{n+1}$)

becomes a covering space.
Note that this space is simply connected because each Z_n is homotopy-equivalent with a finite wedge of 2-spheres (which is simply connected), and given any loop $\sigma : I \to Z$, $\sigma(I)$ is compact and must be contained in Z_n for some n.