MAT4530 (2021 SPRING) MANDATORY ASSIGNMENT

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Typeset your solutions to these problems by LaTeX and submit the file on the course `canvas` page. Due date is March 28.

Problem 1 (Section 1.1, Exercise 5)**.** Let X be a topological space. Show that the following conditions are equivalent.

(1) Any loop $f\colon S^1 \to X$ is homotopic to a constant map.
(2) Any loop $f\colon S^1 \to X$ can be extended to a map $f\colon D^2 \to X$ up to the identification $S^1 = \partial D^2$.
(3) For any $x_0 \in X$, the fundamental group $\pi_1(X, x_0)$ is trivial.

Problem 2 (Section 1.2, Exercise 9)**.** Here M_g is the closed oriented surface of genus g, and M'_g is the complement of an open disk in M_g (punctured surface). Let C be a cycle in M_g that separates M_g to punctured surfaces M'_h and M'_k, with $\partial M'_h = C = \partial M'_k$.

(1) Show that M'_h does not retract onto C.
(2) Show that M_g retracts onto the 'meridian' cycle C' along any hole. (Picture proof is enough.)

Problem 3 (Section 1.3, Exercises 1 and 2)**.** Here $p\colon \tilde{X} \to X$ denotes a covering.

(1) Given $p\colon \tilde{X} \to X$ and a subspace $A \subset X$, show that $\tilde{A} = p^{-1}(A)$ becomes a covering over A.
(2) Given $p_1\colon \tilde{X}_1 \to X_1$ and $p_2\colon \tilde{X}_2 \to X_2$, show that the direct product space $\tilde{X}_1 \times \tilde{X}_2$ becomes a covering over $X_1 \times X_2$.
(3) Use these to show that, given $p_1\colon \tilde{X}_1 \to X$ and $p_2\colon \tilde{X}_2 \to X$, the fiber product space $\tilde{X}_1 \times_X \tilde{X}_2$ becomes a covering over X.

Problem 4 (Section 2.1, Exercise 3)**.** (1) Construct a Δ-complex structure on S^n whose vertices are the points $(0, \dots, \pm 1, \dots, 0) \in S^n \subset \mathbb{R}^{n+1}$.
(2) Construct a Δ-complex structure on $\mathbb{R}P^n$ as a quotient of this structure.

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