homomorphism $F \to R$ is obtained in this way. (Make use of an element $f \geq 0$ in F such that each $f^{-1}[0, c]$ is compact.) Thus the smooth manifold M is completely determined by the ring F. For $x \in M$, show that any R-linear mapping $X : F \to R$ satisfying $X(fg) = X(f)\,g(x) + f(x)\,X(g)$ is given by $X(f) = Df_x(v)$ for some uniquely determined vector $v \in DM_x$.