Problem 2-E. *Isometry theorem.* Let μ and μ' be two different Euclidean metrics on the same vector bundle ξ. Prove that there exists a homeomorphism $f : E(\xi) \to E(\xi)$ which carries each fiber isomorphically onto itself, so that the composition $\mu \circ f : E(\xi) \to R$ is equal to μ'. [Hint: Use the fact that every positive definite matrix A can be expressed uniquely as the square of a positive definite matrix $\sqrt{A}$. The power series expansion

$$\sqrt{(tI + X)} = \sqrt{t}(I + \frac{1}{2t}X - \frac{1}{8t^2}X^2 + - \ldots),$$

is valid providing that the characteristic roots of $tI + X = A$ lie between 0 and 2t. This shows that the function $A \mapsto \sqrt{A}$ is smooth.]

Problem 2-F. As in Problem 1-C, let F denote the algebra of smooth real valued functions on M. For each $x \in M$ let I_x^{r+1} be the ideal consisting of all functions in F whose derivatives of order $\leq r$ vanish at x. An element of the quotient algebra F/I_x^{r+1} is called an r-*jet* of a real valued function at x. (Compare [Ehresmann, 1952].) Construct a locally trivial "bundle of algebras" $\mathcal{Q}_M^{(r)}$ over M with typical fiber F/I_x^{r+1}.