Here are five problems for the reader.

Problem 4-A. Show that the Stiefel-Whitney classes of a Cartesian product are given by

$$w_k(\xi \times \eta) = \sum_{i=0}^{k} w_i(\xi) \times w_{k-i}(\eta) .$$

Problem 4-B. Prove the following theorem of Stiefel. If $n + 1 = 2^r m$ with m odd, then there do not exist 2^r vector fields on the projective space P^n which are everywhere linearly independent.*

Problem 4-C. A manifold M is said to admit a field of tangent k-planes if its tangent bundle admits a sub-bundle of dimension k. Show that P^n admits a field of tangent 1-planes if and only if n is odd. Show that P^4 and P^6 do not admit fields of tangent 2-planes.

Problem 4-D. If the n-dimensional manifold M can be immersed in $\mathbf{R}^{n+1}$ show that each $w_i(M)$ is equal to the i-fold cup product $w_1(M)^i$. If P^n can be immersed in $\mathbf{R}^{n+1}$ show that n must be of the form $2^r - 1$ or $2^r - 2$.

Problem 4-E. Show that the set $\mathfrak{N}_n$ consisting of all unoriented cobordism classes of smooth closed n-manifolds can be made into an additive group. This *cobordism group* $\mathfrak{N}_n$ is finite by 4.11, and is clearly a module over $\mathbf{Z}/2$. Using the manifolds $P^2 \times P^2$ and P^4, show that $\mathfrak{N}_4$ contains at least four distinct elements.

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* Compare [Stiefel, 1936], [Steenrod and Whitehead], [Adams, 1962].