MAT4540/9540 MANDATORY ASSIGNMENT

Choose one of the following topics and prepare a presentation of it for the class on **Tuesday October 15**. The presentation should be between 20 and 30 minutes long.

You may also choose a topic yourself.

Topic 1. Prove the universal coefficient theorem in cohomology. See e.g., [Hat02, §3.1].

Topic 2. If X is a CW complex and G an abelian group, then for each $n > 0$ there is a natural bijection

$$T \colon [X, K(G,n)] \to H^n(X,G)$$

between the homotopy classes of maps from X to $K(G,n)$ and the n-th cohomology of X with coefficients in G. In other words, Eilenberg–Mac Lane spaces represent cohomology theories.

Give the construction of the map T above as well as a sketch of the proof of this result. Consult for example [Hat02, §4.3].

Topic 3. Determine the cohomology ring $H^*(K(\mathbf{Z},n),\mathbf{Q})$ by using the Leray–Serre spectral sequence for the pathspace fibration. See Example 5.7 of `http://www.gradmath.org/wp-content/uploads/2017/12/diaz-final-2017.pdf`.

Topic 4. Define the stable homotopy groups $\pi_i^s(X)$ of a CW complex X. Introduce spectra and discuss the notion of brave new rings. Possible sources are [Hat02, p. 384] and `http://folk.uio.no/rognes/papers/sphere.pdf`.

Topic 5. Sketch the proof of the Dold–Thom theorem `http://people.brandeis.edu/~tbl/dold-thom.pdf`.

References

[Hat02] Allen Hatcher. *Algebraic topology.* Cambridge University Press, Cambridge, 2002, pp. xii+544.