MANDATORY ASSIGNMENT MAT4540 - FALL 2023

The mandatory assignment in MAT4540/9540, fall 2023, will consist of an **around 20 minutes long oral presentation in class**. Please pick one of the topics below for your presentation. If you have a different topic (related to cohomology or homotopy theory) that interests you and that you would rather talk about, feel free to pick that instead. Please let me know by email or in class which topic you choose (so that I know how many will give a presentation).

We plan to have the presentations in **week 45**, i.e., on November 6 and 8. But you may also give your presentation before or after that, just let me know and we'll figure it out.

If you are not able to come to class to give your presentation, please send me an email as soon as possible.

Topic 1 (Steenrod squares, §4.L in Hatcher)**.**

(a) Define Steenrod squares ans Steenrod powers, and list some of their main properties.
(b) Compute $Sq^i(\alpha^n)$ for α a generator of $H^1(\mathbb{R}P^m; \mathbb{Z}/2)$.
(Voluntary) If time permits, give an outline of Example 4L.4 in Hatcher, stating that the degree of a map $f\colon \mathbb{H}P^\infty \to \mathbb{H}P^\infty$ is always a square.

Topic 2 (lim-$\lim^1$, §3.F in Hatcher)**.**

(a) Define the $\lim^1$-functor and give an example of a nontrivial $\lim^1$-group.
(b) Formulate Theorem 3F.8 in Hatcher and give a sample application, for instance Example 3F.9 or 3F.10.

Topic 3 (Transfer maps, §3.G in Hatcher)**.**

(a) For a finitely sheeted covering space $\pi\colon \widetilde{X} \to X$, explain the construction of the *transfer map*
$$\tau\colon C_*(X) \to C_*(\widetilde{X})$$
and the induced maps on (co)homology.
(b) Discuss Proposition 3G.1 and/or Example 3G.2 in Hatcher.

Topic 4. Let $M = (S^2 \times S^1)\#(S^2 \times S^1)$ be the connected sum of two copies of $S^2 \times S^1$.

(a) Explain that M is orientable. Determine the homology groups $H_*(M)$ and the cohomology groups $H^*(M)$ of M.
(b) What can you say about the cup products on the cohomology groups of M?

Topic 5 (Cohomology and $K(G,n)$)**.** Outline a proof sketch that
$$\widetilde{H}^n(X;G) \cong [X, K(G,n)],$$
where the right hand side means homotopy classes of maps from X to the Eilenberg–Maclane space $K(G,n)$. See e.g., Chapter 22, §2 of
`https://www.math.uchicago.edu/~may/CONCISE/ConciseRevised.pdf`

Topic 6 (The Dold–Thom theorem, §4.K in Hatcher)**.** State and give an outline of the proof the Dold–Thom theorem, Theorem 4K.6 in Hatcher.