

MAT 4590-2013
MANDATORY ASSIGNMENT

Deadline: *You must turn in your paper before Thursday, April 18., 2013, 2.30 p.m. in the specially designated box in the 7th floor. Remember to use the official front page available at*

<http://www.mn.uio.no/math/studier/admin/obligatorisk-innlevering/obligforside-eng.pdf>

If you due to illness or other circumstances want to extend the deadline, you must apply for an extension to Robin Bjørnetun Jacobsen (room B718, NHA, e-mail:studieinfo@math.uio.no, phone 22 85 59 07). Remember that illness has to be documented by a medical doctor! See

<http://www.mn.uio.no/math/studier/admin/obligatorisk-innlevering/obligregelverk-eng.html>

for more information about the rules for mandatory assignments

Instructions: *The assignment is compulsory, and students who do not get their paper accepted, will not get access to the final exam. Students who do not get their original paper accepted, but who have made serious and documented attempts to solve the problems, will get one chance of turning in an improved version.*

Problem 1

(a) Let $f : M \rightarrow N$ be a diffeomorphism between two differentiable manifolds. Let $X \in \mathfrak{X}(M)$. Define $f_*(X) \in \mathfrak{X}(N)$ by $f_*(X)(f(p)) = df_p(X(p))$. Let $g \in \mathfrak{D}(M)$. Show that $f_*(gX) = g \circ f^{-1}f_*(X)$.

(b) Let M, N and f be as in (a). Let ∇ an affine connection on M . Define $\nabla' : \mathfrak{X}(N) \times \mathfrak{X}(N) \rightarrow \mathfrak{X}(N)$ by

$$\nabla'_{X'}Y' = f_*(\nabla_{f_*^{-1}(X')}f_*^{-1}(Y')).$$

Show that that ∇' is an affine connection on N .

(c) Now let M and N be Riemannian manifolds with metrics $\langle \cdot, \cdot \rangle$ and $\langle \cdot, \cdot \rangle'$ respectively. Let $f : M \rightarrow N$ be an isometry and ∇ the Riemannian connection on M . Let ∇' be the affine connection on N defined in (b). Show that ∇' is the Riemannian connection on N

Moreover:

Do Carmo:

Chapter 3, Problem 7, 8 and 9.

Chapter 4, Problem 2 and 3.

Chapter 5: Problem 8.

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