Geometry and Analysis, Fall 2018

Problem sheet 3, to be discussed Friday the 21th September.

Problem 1. Let $E \to M$ be a vector bundle equipped with a connection ∇. Show that the section I of $\mathrm{End}(E)$ corresponding to the identity map $E \to E$ is parallel with respect to the induced connection $\tilde{\nabla}$ in $\mathrm{End}(E)$, i.e. $\tilde{\nabla} I = 0$.

Problem 2. Let E, E', E'' be vector bundles over M equipped with connections $\nabla, \nabla', \nabla''$, respectively, and

$$B : E \otimes E' \to E''$$

a bundle homomorphism. For $\phi \in \Omega^k(M; E)$ and $\psi \in \Omega^l(M; E')$ we define $\phi \dot\wedge \psi \in \Omega^{k+l}(M; E'')$ by combining the wedge product with B. Show that if the equation

$$d^{\nabla''}(\phi \dot\wedge \psi) = (d^{\nabla}\phi) \dot\wedge \psi + (-1)^k \phi \dot\wedge (d^{\nabla'}\psi)$$

holds when $k = l = 0$ then it holds in general.