Geometry and Analysis, Fall 2018

Problem sheet 4, to be discussed Monday the 1st October.

Problem 1. Let E, E' be vector bundles over M equipped with connections ∇, ∇', respectively, and let $\tilde{\nabla}$ be the induced connection in $E \otimes E'$. Let $F, F', \tilde{F}$ be the curvatures of $\nabla, \nabla', \tilde{\nabla}$. Show that for vector fields X, Y on M and sections s, t of E, E' one has

$$\tilde{F}(X,Y)(s \otimes t) = F(X,Y)s \otimes t + s \otimes F'(X,Y)t.$$

Problem 2. Let L, L' be complex line bundles over M. Show that

$$c_1(L \otimes L') = c_1(L) + c_1(L').$$

Problem 3. Let $L \to M$ be a complex line bundle. Show that $c_1(L) = 0$ if and only if L admits a flat connection.