Geometry and Analysis, Fall 2018

Problem sheet 5, to be discussed Friday the 9th November.

Problem 1. Let A be any set and $c : A \to \mathbb{C}$ a function with

$$\lim_{a\to\infty} c(a) = 0$$

in the sense explained in the notes. Show that the operator

$$\ell^p(A) \to \ell^p(A), \quad f \mapsto cf$$

is compact for $1 \leq p < \infty$.

Problem 2. Let M be a Riemannian manifold and E, E' Hermitian vector bundles over M. Let $L : \Gamma(E) \to \Gamma(E')$ be a differential operator with injective symbol, i.e. for all $p \in M$ and non-zero $\xi \in T_p^*M$ the symbol $\sigma(L,p)\xi$ is injective. Show that L^*L is elliptic.

Problem 3. Show that on any Riemannian manifold M the Laplacian $d^*d : C^\infty(M) \to C^\infty(M)$ is elliptic.