EXAM in MAT4720/9720 Spring 2020

The oral exam will last about 30/40 minutes and it will be held at the blackboard. During exams a series of questions on the material we have gone through during the course will be asked. Both statements and proofs (when done in class) are in the program. Also some exercises could be asked in the same form that they were presented in class (see examples, exercise sessions, proofs of simple results that were often used as applications of other major results).
No material is going to be available, but hints will be provided when necessary.

A piece of advice, when presenting the proofs, particularly when long and articulated, it is beneficial to give an overview of the argument, possibly in steps. This is often easier to have it in place, then the details can be filled in.

As for the exercises, it is beneficial if one presents the possible strategy, as a way to share the mathematical thinking.

For your best use, hereafter you find a list of possible questions.

1) Stochastic processes, different forms of measurability with respect to an information flow (i.e. filtrations). Some comments on the importance of progressive measurability, relative results
2) Kolmogorov existence theorem for the construction of a stochastic process from finite dimensional distributions
3) Brownian motion with respect to its natural filtration and a general filtration
4) Kolmogorov continuity theorem (statement) and use, for instance, in the case of Brownian motion
5) Brownian motion as a Gaussian process and characterisation of Gaussian processes
6) Brownian motion and the variation of the paths. Proof of the result in the L^2 setting. Statement of the Lévy theorem
7) Martingales (sub-, super-), Doob decomposition theorem in discrete time, convergence theorem P-a.s.; bounded martingales in L^p and convergence P-a.s. and in L^p; uniform integrability; Convergence in L^1 and uniform integrability.
8) Martingales in continuous time and results on convergence. Some relevant inequalities.
9) Exponential martingales and characterisation of Brownian motion. Girsanov's theorem and change of measures. Stochastic dynamics under change of measures
10) Doob decomposition in continuous time, the predictable compensator
11) Construction of the stochastic integral in the L^2 setting.
12) Stopping times, Doob optional sampling theorem.
13) Local martingales, properties and some relevant connections to martingales, sub-, super
14) Construction of the stochastic integral in the extended form
15) Ito formula
16) Multidimensional ito formula and product rule; solution of a linear differential equation
17) Stochastic integral representation theorem
18) Martingale representation theorem
19) Existence and uniqueness different concepts for stochastic differential equations
20) Sufficient conditions for the existence of a strong solution and its strong uniqueness