EXAM in MAT4720/9720 Spring 2022

The oral exam will last about 30/40 minutes and it will be held at the blackboard. During the exam a series of questions on the course material will be asked. This may include statements and proofs (when done in class) of the results presented and applications of these in form of short exercises. No material or notes are admitted, but hints will be provided when necessary.

Some advice.
When presenting the proofs, particularly when long and articulated, it is beneficial to give an overview of the argument, possibly in steps. This is often easier to have it in place, then the details can be filled in.
As for the exercises, it is beneficial if one presents/discusses the possible strategy, as a way to share the mathematical thinking.

For the program, please refer to the webpage of the course under Schedule.

For your best use, hereafter you find a list of possible questions.

1) Stochastic processes, different forms of measurability with respect to an information flow (i.e. filtrations). Some comments on the importance of progressive measurability, relative results
2) Kolmogorov existence theorem (statement) for the construction of a stochastic process from finite dimensional distributions
3) Brownian motion with respect to its natural filtration and a general filtration
4) Kolmogorov continuity theorem (statement) and use, for instance, in the case of Brownian motion
5) Brownian motion and the variation of the paths. Proof of the result in the L^2 setting. Statement of the Lévy theorem
6) Martingales (sub-, super-), Doob decomposition theorem in discrete time, convergence theorem P-a.s.; bounded martingales in L^p and convergence P-a.s. and in L^p; uniform integrability; Convergence in L^1 and uniform integrability. Proof of the results presented
7) Martingales in continuous time and results on convergence. Some relevant inequalities.
8) Doob decomposition in continuous time, the predictable compensator
9) Construction of the stochastic integral in the L^2 setting with proof of the arguments presented
10) Stopping times, Doob optional sampling theorem.
11) Local martingales, properties and some relevant connections to martingales, sub-, super with proof of the results presented
12) Construction of the stochastic integral in the extended form
13) Ito formula (statement and use)
14) Existence and uniqueness different concepts for stochastic differential equations
15) Sufficient conditions for the existence of a strong solution and its strong uniqueness (proof)

16) Numerical methods for simulation, Eurler scheme with proof of the results presented.
17) Multidimensional Ito formula and product rule; solution of a linear differential equation
18) Stochastic integral representation theorem (proof)
19) Martingale representation theorem (proof)
20) Exponential martingales and characterisation of Brownian motion. Girsanov's theorem (proof) and change of measures. Stochastic dynamics under change of measures