MAT4740/9740
Malliavin calculus and applications to finance
Exam.

The exam is held at the blackboard for a duration of about 30-40 minutes.

It is organized in two parts. In the first 15 minutes the candidate is asked to present a topic of his choice selected among the ones here below marked with *.
The candidate can organize his/her reply bearing in mind to:
- Center the topic and address the major results
- Be prepared on the proofs of the statements, as presented/discussed in class or left as exercise after discussion
- Be prepared on the notions that are embedded in the topic addressed.

In the second part, questions from the program will be asked. This includes the use of notions and techniques acquired in some specific context, e.g. short exercise.

List of topics, which are going to be tested during the exam.

PART I - Brownian motion
1. Itô integration and construction of iterated Itô integrals, multiple Itô integrals, their properties, connection with Hermite polynomials. Wiener chaos decomposition (a.k.a. Itô chaos expansion).
2. Malliavin derivative: construction from smooth random variables. Representation of the Malliavin operator via chaos expansions.
3. Closeness of linear operators. Malliavin derivative operator on smooth randomvariables is closable.
4. Skorohod integral and chaos expansion representation, dual operator of the Malliavin derivative, duality and integration by parts, fundamental theorem of calculus. relationship between Skorohod integral and Itô integral.
5. Itô represenation theorem and the Clark-Haussmann-Ocone formula. Chain rule, Clark-Haussmann-Ocone formula under change of measure,their application to hedging in a complete market driven by Brownian motion.
6. Application to sensitivity analysis: computation of the Delta. Specific application of Malliavin methods.
7. Existence of density of probability laws and their properties: the problem, the use of integration by parts and duality, the use of Malliaivn caluculus in this context.

PART II - Centered Poisson random measure
1. Lévy processes, infinitely divisible distribution and the Lévy-Khintchine formula, Lévy measure and Poisson random measure associated to a Lévy process.
2. Lévy-Itô processes, Itô formula for Lévy-Itô processes, Lévy-Itô decomposition theorem; Itô representation theorem with respect to the Poisson random measure.
3. Combination of the Brownian and Poisson structures: product spaces; mixtures of Gaussian and Poisson random measures.

4. Iterated Itô integrals and chaos expansions with respect to the centered Poisson random measure.
5. Malliavin derivative and Skorohod integral with respect to the centered Poisson random measure, the operators, duality, integration by parts, fundamental theorem of calculus. Relationship between Skorohod integral and Itô integral.
6. Chain rule for the Malliavin Derivative with respect to the Poisson random measure.
7. Itô Integral representation theorem and Clark-Haussmann-Ocone formula with respect to the centered Poisson random measure. Combination with the Itô integral representation with respect to the mixture of Gaussian and centered Poisson random measure, Clark-Haussmann-Ocone formula in this last case.
8. Non-anticipating derivative and representation theorems.
9. Application to minimal variance hedging in incomplete markets driven by Gaussian and centred Poisson noises.
10. Computation of the sensitivity parameters to model risk in dynamics driven by Gaussian and centered Poisson random measure: focus on the Delta. approaches to the computation: the density method, the conditional density method, the Malliavin method.

(*) Topics for presentations:

1. Stochastic derivatives and integrals: duality formulae. Also application to the study of existence of densities for random variables and SDEs.
2. Integral representations and hedging.
3. Sensitivity analysis.