MAT4740/9740
Malliavin calculus and applications to finance
Exam

The exam is oral and lasts about 40 minutes.

It is organized in two parts.
The first part lasts no more than 15-20 minutes (max), and the candidate is presenting a given topic prepared in advance with slides, to be efficient (else on the board if the time and the content are equally well paced).
In preparing the presentation one should bear in mind to:
- Center the topic and address the major results
- Be prepared on the notions that are embedded in the topic addressed.

The topics for presentation are:

a. Product rules and chain rules in the Brownian and Poisson framework
b. Chain rules and the computation of the delta in sensitivity analysis Brownian framework.
c. Fundamental theorem in Malliavin calculus and the computation of the delta in the Brownian framework

The second part of the exam is left for questions on the program.
This includes the use of notions and techniques acquired in some specific context.

Here below you find a list of topics, which are going to be tested during the exam.

Brownian motion framework:

1. Itô integration and construction of iterated Itô integrals, multiple Itô integrals, their properties, connection with Hermite polynomials. Wiener chaos decomposition (a.k.a. Itô chaos expansion).
2. Malliavin derivative via chaos expansion approach.
3. Closedness of the operator
4. Skorohod integral and chaos expansion representation, dual operator of the Malliavin derivative, duality and integration by parts, fundamental theorem of calculus. Relationship between Skorohod integral and Itô integral.
5. Itô represenation theorem and the Clark-Haussmann-Ocone formula. Chain rules. Product rule.
6. Application to sensitivity analysis: computation of the Delta. Specific application of Malliavin methods.
7. Functional analytic approach to the construction of Malliavin operators and the relationship with the Malliavin derivative via chaos expansion.
8. Forward integration and relationship with the Malliavin/Skorohod calculus as well as with the Ito calculus

Framework for the centered Poisson random measure:

1. Lévy processes, infinitely divisible distribution and the Lévy-Khintchine formula, Lévy measure and Poisson random measure associated to a Lévy process.
2. Lévy-Itô processes, Itô formula for Lévy-Itô processes, Lévy-Itô decomposition theorem; Itô representation theorem with respect to the Poisson random measure.
3. Iterated Itô integrals and chaos expansions with respect to the centered Poisson random measure.
4. Malliavin derivative and Skorohod integral with respect to the centered Poisson random measure, the operators, duality, integration by parts, fundamental theorem of calculus. Relationship between Skorohod integral and Itô integral.
5. Product rule. Chain rule for the Malliavin Derivative with respect to the Poisson random measure.
6. Itô Integral representation theorem and Clark-Haussmann-Ocone formula with respect to the centered Poisson random measure.

Mixed framework:

1. Description of the mixture of measures vs the product of spaces
2. Clark-Ocone formula in the mixed case