MAT4750 - Mathematical Finance: modelling and risk management

ASSIGNMENT

Evaluation: passed/not passed.

About subordination.

This exercise is all about a subclass of Lévy processes, which is know as subordinators. These are often used as stochastic time-change, in simulation techniques, and they also find application in stochastic volatility modelling. See, e.g., the Barndorff-Nielsen and Shephard (BNS) model. The main reference for this assignment is the book by Cont and Tankov, see below. You can also find other material within the course literature.

1. Give a definition of subordinator.

2. Give the characteristic function (Lévy-Khintchin fromula) and the characteristic triplet of a subordinator.

3. Using the points above, show that the Poisson process is a subordinator.

4. Prove the following satement: *Let $X = X_t$, $t \geq 0$, be a Lévy process. Define*

$$S_t := \sum_{s \leq t; \Delta X_s \neq 0} (\Delta X_s)^2, \quad t \geq 0.$$

The process above is a subordinator. See [CT, Proposition 3.11].

5. Let $B = B_t$, $t \geq 0$, be a Brownian motion and $S = S_t$, $t \geq 0$, a subordinator independent of B. Consider the process

$$X_t := B_{S_t}, \qquad t \geq 0.$$

This is called *subordinated Brownian motion* or *time-changed Brownian motion.* This process is a Lévy process. Describe its characteristic function and characteristic triplet following [CT, Theorem 4.2].

6. The BNS stochastic volatility model is based on a linear stochastic differential equation driven by a subordinator Z:

$$dV_t = -\lambda V_t dt + Z_t; \qquad V_0 > 0 \qquad (\lambda > 0).$$

Using the Itô formula and the Lévy-Itô representation, verify that the solution of this equation is given by

$$V_t = e^{-\lambda t} V_0 + \int_0^t e^{\lambda(s-t)} dZ_s.$$

You can read about this model in [CT, Section 15.3 and onward].

References

[CT] Rama Cont and Peter Tankov: *Financial Modelling with Jump Processes*, Chapman and Hall 2004.