MAT4760 Spring semester 2012
Infinite-Dimensional Stochastic Dynamical Systems

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Lectures
Mondays 12.15 - 14.00 in seminar room B91
Tuesdays 10.15 -12.00 in seminar room B70
First lecture Monday 23 January 2012.

Abstract

The course is designed to present a dynamical viewpoint of infinite-dimensional stochastic systems. We will focus on two large classes of stochastic differential systems: *stochastic systems with memory* and *quasilinear stochastic partial differential equations*. Topics to be discussed are outlined below (as much as time permits).

I. Stochastic Systems with Memory

- Examples of stochastic functional differential equations (sfde's). Existence, uniqueness and regularity of solutions to sfde's.
- Average dynamics: Markov behavior and the infinitesimal generator.
- Pathwise dynamics: regular and singular sfde's.
- Construction of infinite-dimensional cocycles for regular sfde's.
- Asymptotics and the Lyapunov spectrum for linear and nonlinear sfde's.

II. Stochastic Partial Differential Systems

- Mild solutions of semilinear/quasilinear stochastic partial differential equations (spde's).
- Linear parabolic spde's: Construction of the semiflow. The Lyapunov spectrum.
- Semilinear spde's: Existence and regularity of the stochastic semiflow.
- Construction of infinite-dimensional cocycles for quasilinear spde's.
- Examples: Stochastic Burgers equation; $2D$ stochastic Navier-Stokes equations (SNSE's). Dynamics and stability.

References

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