14th February, 2019

MAT4770/9770

Mandatory assignment 1 of 1

Submission deadline

Thursday 14th March 2019, 14:30 at Devilry (devilry.ifi.uio.no).

Instructions

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. Students who fail the assignment, but have made a genuine effort at solving the exercises, are given a second attempt at revising their answers. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) well before the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

All exercises must be satisfactory answered to pass the assignment.

In this exercise we will analyse a structured deal between to parties in the power market. The deal is analogous to the agreement between Aquila and Consolidated Edison (ConEd) in 1996,[1] where ConEd bought power from Aquila with a temperature-dependent rebate on the price.

Suppose that Aquila delivers power in the period $[\tau_1, \tau_2]$, that is, delivers $\int_{\tau_1}^{\tau_2} S(u)du$ where we denote $S(u)$ the spot power price at time u. Denote the price that ConEd pays for this at time $t \leq \tau_1$, when entering the deal, by $F_{\text{ConEd}}(t, \tau_1, \tau_2)$. This is the price that ConEd pays at time τ_2 in return of receiving the power, if there is *no rebate*. However, there is an additional clause included in the deal that says that if the $\text{CDD}(\tau_1, \tau_2)$ [2] for the period $[\tau_1, \tau_2]$ is below a threshold κ, then a discount of $d \in (0, 1)$ on $F_{\text{ConEd}}(t, \tau_1, \tau_2)$ is given.

Problem 1. Argue that Aquila has the payoff

$$(F_{\text{ConEd}}(t, \tau_1, \tau_2) - F_{\text{ConEd}}(t, \tau_1, \tau_2) \times d \times 1(\text{CDD}(\tau_1, \tau_2) < \kappa)) - \int_{\tau_1}^{\tau_2} S(u)du$$

from the deal. Here, $1(\cdot)$ is the indicator function.

Explain why ConEd wanted a rebate if the CDD was lower than a threshold, in view of the fact that the actual contract was entered for the summer month of August with the CDD measured in New York.

Problem 2. Show that

$$F_{\text{ConEd}}(t, \tau_1, \tau_2) = \frac{F(t, \tau_1, \tau_2)}{1 - d\mathbb{E}[1(CDD(\tau_1, \tau_2) < \kappa)|\mathcal{F}_t]}$$

where $F(t, \tau_1, \tau_2)$ is the forward price on power

$$F(t, \tau_1, \tau_2) = \mathbb{E}[\int_{\tau_1}^{\tau_2} S(u)du|\mathcal{F}_t]$$

and $\mathcal{F}_t$ is the given filtration. Notice that we choose the pricing measure here to be the market probability.[3] Show that the price that ConEd actually pays will either be more expensive than the the power forward price, or less expensive.

[1] See "weather derivative" on Wikipedia

[2] CDD is the 'cooling degree day index'

[3] I.e., no change of probability; $Q = P$!

Problem 3. Download the temperature data from the web-page of MAT4770, and create a plot of the time series. For the given temperature data, estimate a seasonal CAR(p)-model.[4] You can assume that the volatility is constant in this model.

For the month of August, compute the CDD-index for every year based on the given data and derive the mean value for the CDD-index.[5] The threshold in the CDD-index is set to 18°C. Denote the mean you find by κ. Assuming we are at time t being July 1, based on your temperature model calculate $\mathbb{E}[1(CDD(\tau_1, \tau_2) < \kappa)|\mathcal{F}_t]$ for the month of August. In the CAR-model, you can suppose that $X_i(t) = 0$ for $t = 1$July, $i = 1, 2, 3, .., p$. Recall that $X_1(t)$ is the de-seasonalized temperature, and the value zero means that the temperature is equal to the seasonal average.

Problem 4. Download the forward price power data from the web-page of MAT4770, and create a plot of the time series. These data are proxies for front-month delivery forward prices, and we use them for estimating the volatility and drift in a geometric Brownian motion dynamics

$$\frac{dF(t, \tau_1, \tau_2)}{F(t, \tau_1, \tau_2)} = \mu dt + \sigma dB(t)$$

Estimate μ and σ from the data. Notice that we completely ignore the Samuelson effect here, but set the volatility to a constant.

Problem 5. In the US, it is natural to think that summer power prices are positively correlated with temperature, that is, the warmer it gets, the higher power prices we see, and vice versa. Why do you think this is the case?

Assume that the Brownian motion driving the power forward is correlated 0.9 with the Brownian motion driving your temperature CAR(p)-dynamics. Develop an algorithm that simulates day-by-day the temperature and the forward price $F(t, \tau_1, \tau_2)$. Use this to generate 10 scenarios of $F_{\text{ConEd}}(t, \tau_1, \tau_2)$ for t ranging from 1 July to 31 July, and $[\tau_1, \tau_2]$ is the month of August. As starting value of $F(t, \tau_1, \tau_2)$ you can use US\$31600[6]. From these scenarios, comment on how the price $F_{\text{ConEd}}(t, \tau_1, \tau_2)$ at 1 July, when contract was entered, changes with time through July. For each scenario, also simulate the temperature through August, and find out if the rebate will be triggered or not. On September 1, what will Consolidated Edison eventually pay

[4] Find a suitable p, the order of autoregression

[5] Here, $[\tau_1, \tau_2]$ is the month of August

[6] The forward prices are for the total delivery, and not per MWh as is the more common market quotation.

in each scenario? You are free to choose what software you use for the analysis.[7]

Notice that the temperature and forward price data are not collected from the New York area, and thus are not directly comparable for the deal between Aquila and Consolidated Edison. However, the price and temperatures are real, so they give a realistic picture at least.

[7] Excel, Matlab, R, Python etc....