11th February, 2020

COURSE MAT4770

Mandatory assignment 1 of 1

Submission deadline

Thursday 5th March 2020, 14:30 in Canvas (canvas.uio.no).

Instructions

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. Students who fail the assignment, but have made a genuine effort at solving the exercises, are given a second attempt at revising their answers. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) well before the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

In answering the numerical parts of this assignment, it is recommended to use Matlab of R. You may also use programming languages as Python.

Problem 1. Assume that the temperature dynamics in a city is given by

$$T(t) = \Lambda(t) + \vec{e}_1^\top \vec{X}(t)$$

where $T(t), t \geq 0$ is the temperature in degrees Celsius at time t, measured in *days*. Furthermore, $\Lambda(t)$ is the seasonality function and $\vec{X}(t)$ is a CAR(3) process with time-varying (deterministic) volatility $\sigma(t)$. In this Exercise you are going to develop a Monte Carlo simulator for the temperature, and analyse CDD futures prices.

For simplicity, we assume $\Lambda(t) = 18$, that is, the seasonality is constant and equal to 18 degrees Celsius. Moreover, assume that the CAR(3)-parameters are given as for Stockholm in Table 10.5 (page 301) in the lecture book. Moreover, we assume σ to be a constant equal to 2. Initial temperatures, whenever you need these, are set equal to 18, and you can use $X_2 = 0 = X_3$ where these two are the second and third coordinates in $\vec{X}$.

Part 1a

Program a simulator that gives you paths of $T(t)$ for any time horizon and time-stepping Δ that you specify. Program two different ways to approximate the paths, one based on naive Euler and another based on exact simulation. The former is based on the approximation

$$\vec{X}(t + \Delta) - \vec{X}(t) \approx A\vec{X}(t)\Delta + \sigma\vec{e}_3\Delta B(t) \tag{1}$$

with $\Delta B(t) = B(t + \Delta) - B(t)$ and time-step Δ. The latter is based on the exact relationship

$$\vec{X}(t + \Delta) = e^{A\Delta}\vec{X}(t) + \sigma \int_t^{t+\Delta} e^{A(t+\Delta-u)}\vec{e}_3 dB(u) \tag{2}$$

Show (2). Approximate the stochastic integral with

$$\int_t^{t+\Delta} e^{A(t+\Delta-u)}\vec{e}_3 dB(u) \approx e^{A\Delta}\vec{e}_3\Delta B(t)$$

and use this in your simulations of the second method.

Show plots where you compare the two methods by simulating paths of the temperature dynamics with daily time-stepping (and using the same Brownian motion simulation, i.e., using the same sequence of iid standard normal when simulating $\Delta B(t)$).

Part 1b

Recall the CDD price formula in Proposition 10.6, page 306, in the lecture book. Implement this formula, where the user can give as input the measurement period of the CDD, the market price of risk (which we assume constant) and current time, and the output is the CDD futures price based on the temperature model from Part (a). You use as parameters for the temperature dynamics the same as above, with threshold $c = 18$. In the integration, use simple Riemann approximation of the integral, with daily sampling. A daily sampling is mimicking the market convention.

Next, implement a Monte Carlo-based simulator for CDD futures pricing, based on the two path simulators in Part (a) above. Analyse the accuracy of the Monte Carlo pricer as a function of number of paths, where you use the exact formula above as benchmark. Base your analysis on a case where you choose the market price of risk equal to zero, current time as zero, a measurement period starting in 10 days from now and lasting for 30 days.

Let the market price of risk be $\theta = \pm 1$. What is then the CDD futures prices using the exact formula and the Monte Carlo based approach? For the Monte Carlo method, you must modify the simulation approaches from Part (a) to allow to a θ. Discuss the risk premium for the two cases of market price of risk.

Problem 2. Let the forward price at time $t \geq 0$ of a contract on a commodity delivering at time $\tau \geq t$, denoted $f(t, \tau)$, be given by

$$\frac{df(t,\tau)}{f(t,\tau)} = \sigma e^{-\alpha(\tau - t)} dW(t)$$

This is the dynamics under a pricing measure Q where W is a Q-Brownian motion. We have that σ and α are two positive parameters.

Part 2a

First, find $f(t, \tau)$, that is, solve its stochastic differential equation. Next, derive the price at time $t \leq T$ of a call option on the forward, where the call has strike K and exercise time $T \leq \tau$. The risk-free interest rate is supposed to be a constant and denoted by r. You have derived the Black-76 formula. Implement this formula, where the price is computed from given inputs $t, \tau, T, f(t, \tau), K, r, \sigma, \alpha$.

Part 2b

In this exercise you are supposed to find the *implied volatilities* from call options on power futures in the EEX-market. For simplicity, we assume in the exercise that $\alpha = 0$ and $r = 0$. We also suppose that current time is $t = 0$.

If you have available a price on a call, denoted $\widehat{C}$, then the implied volatility is defined as the σ you must insert into the Black76-formula in order to recover $\widehat{C}$. The price $\widehat{C}$ is observed in the market for contractually given parameters τ, T and K, so the computed implied volatility will become a function of these three parameters.

Visit eex.com, and go to "Market Data". Under the heading "Power", click on "Options" and further "Phelix DE-options", which leads you to a web-page with baseload option prices written on monthly German futures on power. In the table you see, there will be a list of the various months of the underlying with settlement prices which are the underlying forward price from last trade. If you click on one of these months, you will obtain prices of call options for different strikes. Find implied volatilities for some of the delivery months and plot these as a function of the strike. Comment on your findings.

Notice that the Black76-formula is for fixed-delivery forwards. Please choose the delivery time of the forward in the above task as the middle point of the month of delivery, i.e., if you have delivery in March, choose τ to be March 15th. "Option expiry" gives you the exercise time of the option.