Weather Derivatives: Modeling and Pricing

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The temperature market

The temperature market

- Chicago Mercantile Exchange (CME) organizes trade in temperature derivatives:
  - Futures contracts on weekly, monthly and seasonal temperatures
  - European call and put options on these futures
- Contracts on several US, Canadian, Japanese and European cities

HDD, CDD and CAT

- HDD (heating-degree days) over a period $[\tau_1, \tau_2]$

$$\int_{\tau_1}^{\tau_2} \max(18 - T(u), 0)\, du$$

- HDD is the accumulated degrees when temperature $T(u)$ is below 18°C
- CDD (cooling-degree days) is correspondingly the accumulated degrees when temperature $T(u)$ is above 18°C
- CAT = cumulative average temperature
  - Average temperature here meaning the *daily* average

$$\int_{\tau_1}^{\tau_2} T(u)\, du$$

At the CME...

- Futures written on HDD, CDD, and CAT as index
  - HDD and CDD is the index for US temperature futures
  - CAT index for European temperature futures, along with HDD and CDD
- Discrete (daily) measurement of HDD, CDD, and CAT
- All futures are cash settled
  - 1 trade unit=20 Currency (trade unit being HDD, CDD or CAT)
  - Currency equal to USD for US futures and GBP for European
- Call and put options written on the different futures

The wind market

- The US Futures Exchange launched wind futures and options summer 2007
- Futures on a wind speed index (Nordix) in two wind farm areas
    - Texas and New York
    - Texas divided into 2 subareas, New York into 3
- The Nordix index aggregates the daily *deviation* from a 20 year mean over a specified period
    - Benchmarked at 100
- Futures are settled against this index
    - European calls and puts written on these futures

- Formal definition of the index:

$$N(\tau_1, \tau_2) = 100 + \sum_{s=\tau_1}^{\tau_2} W(s) - w_{20}(s)$$

- $W(s)$ is the wind speed on day s
  - Daily average wind speed
  - Typically measured at specific hours during a day
- $w_{20}(s)$ is the *20-year average* wind speed for day s
- $[\tau_1, \tau_2]$ measurement period, typically a month or a season

Stochastic models for temperature and wind

A continuous-time AR(p)-process

- Dynamics of daily average wind and temperatures are well-described by autoregressive time series models (AR-models)
- Purpose of pricing derivatives: continuous-time model
  - Futures prices vary over the day.....
- Define the Ornstein-Uhlenbeck process $\mathbf{X}(t) \in R^p$

$$d\mathbf{X}(t) = A\mathbf{X}(t)\,dt + \mathbf{e}_p\sigma(t)\,dB(t)\,,$$

- $\mathbf{e}_k$: k'th unit vector in R^p, $\sigma(t)$ "volatility"
- A: $p \times p$-matrix

$$A = \begin{bmatrix} \mathbf{0} & & \mathbf{I} \\ -\alpha_p & \cdots & -\alpha_1 \end{bmatrix}$$

- Explicit solution of $\mathbf{X}(s)$, given $\mathbf{X}(t)$:

$$\mathbf{X}(s) = \exp\left(A(s-t)\right)\mathbf{X}(t) + \int_t^s \exp\left(A(s-u)\right)\mathbf{e}_p\sigma(u)\,dB(u)\,,$$

- Define a continuous-time AR(p)-process as

$$X_1(t) = \mathbf{e}_1'\mathbf{X}(t)$$

- Basic buidling blocks for describing the temperature and wind dynamics
    - Named a CAR(p)-process
    - Subclass of the CARMA(p,q)-processes

Why is X_1 a CAR(p) process?

- Consider $p = 3$
- Do an Euler approximation of the $\mathbf{X}(t)$-dynamics with time step 1
  - Substitute iteratively in $X_1(t)$-dynamics
  - Use $B(t+1) - B(t) = \epsilon(t)$
- Resulting discrete-time dynamics

$$
\begin{aligned}
X_1(t+3) \approx (3-\alpha_1)X_1(t+2) + (2\alpha_1 - \alpha_2 - 1)X_1(t+1) \\
+ (\alpha_2 - 1 + (\alpha_1 + \alpha_3))X_1(t) + \sigma(t)\epsilon(t)\,.
\end{aligned}
$$

- Empirical analysis suggests the following models for temperature and wind:
- Temperature dynamics $T(t)$ defined as

$$T(t) = \Lambda(t) + X_1(t)$$

- Wind dynamics $W(t)$ defined as (Box-Cox transform)

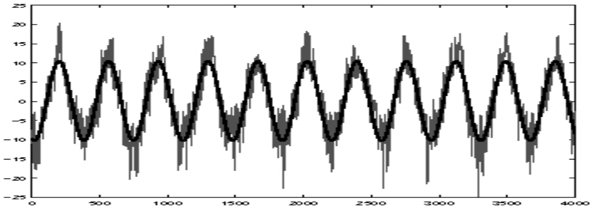
$$W(t) = \begin{cases} (\lambda(\Lambda(t) + X_1(t)) + 1)^{1/\lambda}, & \lambda \neq 0 \\ \exp\left(\Lambda(t) + X_1(t)\right), & \lambda = 0 \end{cases}$$

- $\Lambda(t)$ seasonality function

Empirical analysis of temperature and wind data

Empirical study of Stockholm temperature data

- Daily average temperatures from 1 Jan 1961 till 25 May 2006
  - 29 February removed in every leap year
  - 16,570 recordings
- Last 11 years snapshot with seasonal function



- Fitting of model goes stepwise:
    1. Fit seasonal function $\Lambda(t)$ with least squares
    2. Fit AR(p)-model on deseasonalized temperatures
    3. Fit seasonal volatility $\sigma(t)$ to residuals

1. Seasonal function

- Suppose seasonal function with trend

$$\Lambda(t) = a_0 + a_1\, t + a_2 \cos\left(2\pi(t - a_3)/365\right)$$

- Use least squares to fit parameters
  - May use higher order truncated Fourier series
- Estimates: $a_0 = 6.4, a_1 = 0.0001, a_2 = 10.4, a_3 = -166$
  - Average temperature increases over sample period by 1.6°C

2. Fitting an auto-regressive model

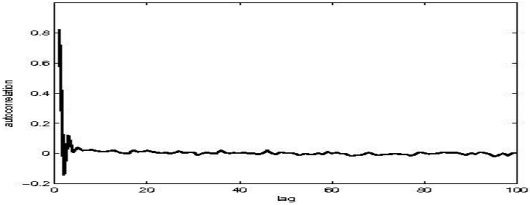
- Remove the effect of $\Lambda(t)$ from the data

$$Y_i := T(i) - \Lambda(i)\,, i = 0, 1, \ldots$$

- Claim that AR(3) is a good model for Y_i:

$$Y_{i+3} = \beta_1 Y_{i+2} + \beta_2 Y_{i+1} + \beta_3 Y_i + \sigma_i \epsilon_i\,,$$

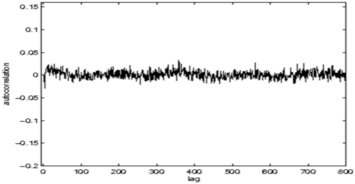
- The partial autocorrelation function for the data suggests AR(3)

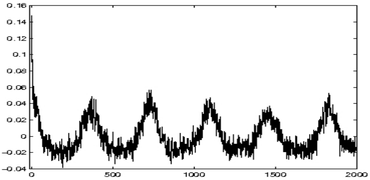


- Estimates $\beta_1 = 0.957, \beta_2 = -0.253, \beta_3 = 0.119$ (significant at 1% level)
- R^2 is 94.1% (higher-order AR-models did not increase R^2 significantly)

3. Seasonal volatility

- Consider the residuals from the AR(3) model
- Close to zero ACF for residuals
- Highly seasonal ACF for *squared* residuals



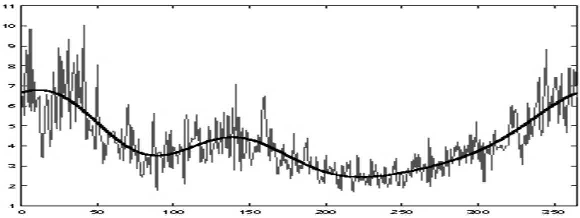


- Suppose the volatility is a truncated Fourier series

$$\sigma^2(t) = c + \sum_{i=1}^{4} c_i \sin(2i\pi t/365) + \sum_{j=1}^{4} d_j \cos(2j\pi t/365)$$

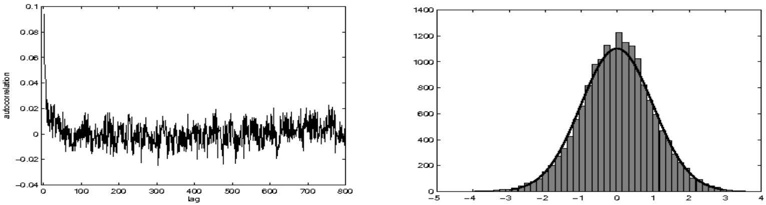
- This is calibrated to the daily variances
  - 45 years of daily residuals
  - Line up each year next to each other
  - Calculate the variance for each day in the year

- A plot of the daily empirical variance with the fitted squared volatility function
- High variance in winter, and early summer
- Low variance in spring and late summer/autumn



- Same observation for other cities (Berlin, US, Norway, Lithuania)

- Dividing out the seasonal volatility from the regression residuals
- ACF for squared residuals non-seasonal
    - ACF for residuals unchanged
    - Residuals become (close to) normally distributed



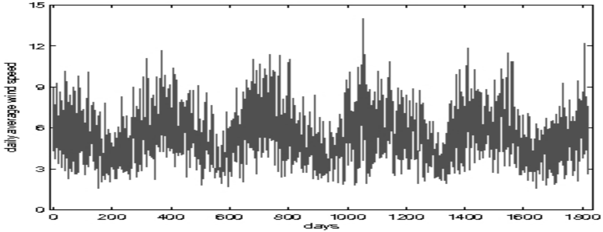
- Conclusion: fitted an AR(3)-model with seasonal variance to deseasonalized daily temperatures
- Apply the link between CAR(3) and AR(3) to derive the continuous-time parameters α_1, α_2 and α_3

$$\alpha_1 = 2.043, \alpha_2 = 1.339, \alpha_3 = 0.177$$

- Seasonality Λ and variance σ given
- The fitted CAR(3)-model is stationary (to a normal distribution)
  - Eigenvalues of A have negative real parts

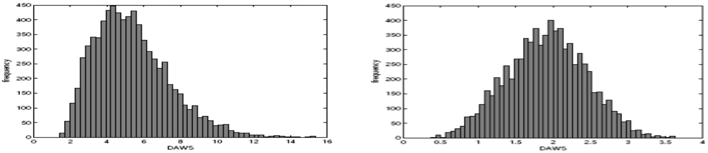
Empirical study of New York wind speed data

- Daily average wind speed data from New York wind farm region 1 from Jan 1 1987 till Sept 7 2007.
- 7,550 daily recordings, after leap year data were removed
- Figure shows 5 years from 1987



- Fitting wind speed model to data follows (almost) the same scheme as temperature
  1. Transform data to symmetrize
  2. Fit seasonal function
  3. Find AR(p)-model to deseasonalized data
  4. Find volatility structure of residuals

1. Symmetrization of data



- Wind speed histogram (left), Box-Cox power transformed speeds (right) with $\widehat{\lambda} = 0.2$
- Box-Cox transform

$$y^{(\lambda)} = \begin{cases} \frac{y^{\lambda}-1}{\lambda}, & \lambda \neq 0 \\ \ln y, & \lambda = 0 \end{cases}$$

2. Seasonal function

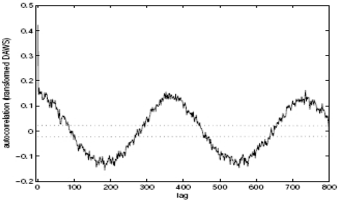
- Seasonality function with annual and biannual periodicity

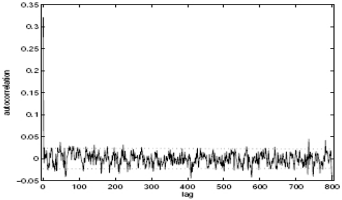
$$\Lambda(t) = a_0 + a_1 \cos(2\pi t/365) + a_2 \sin(2\pi t/365) + a_3 \cos(4\pi t/365) + a_4 \sin(4\pi t/365)$$

- Nonlinear least squares (using matlab) on transformed data gives

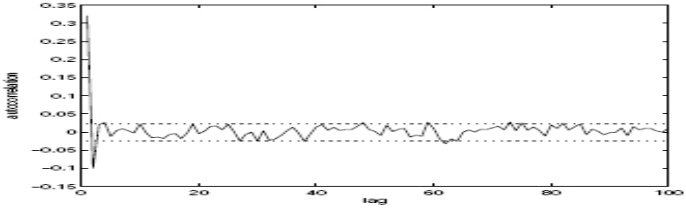
$$a_0 = 1.91, a_1 = 0.26, a_2 = 0.08, a_3 = -0.04, a_4 = -0.07$$

- Consider the ACF *before* and *after* estimated seasonality has been removed
- We see (right plot) that the ACF of deseasonalized data does not show any periodic pattern





3. Fitting an AR(p)-model



- Partial ACF for deseasonalized data suggests a higher-order AR(MA) structure
  - AR(4) best according to Akaike's Information Criterion
  - ...best among ARMA($p \leq 5, q \leq 5$)

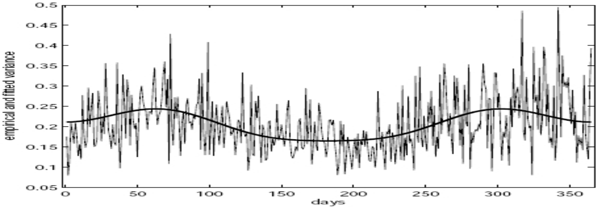
- Estimated regression parameters in the AR(4) model

$$z_t = \beta_1 z_{t-1} + \beta_2 z_{t-2} + \beta_3 z_{t-3} + \beta_4 z_{t-4}$$

$$\beta_1 = 0.355, \beta_2 = -0.104, \beta_3 = 0.010, \beta_4 = 0.027$$

- All except β_3 are found to be significant

4. Volatility structure



- Estimated daily empirical variance, and fitted a truncated Fourier series
  - ...as for temperature

$$\sigma^2(t) = c_0 + \sum_{k=1}^{3} c_k \cos(2\pi kt/365)$$

- Fitting using (nonlinear) least squares in Matlab
- Estimated parameters

$$c_0 = 0.208, c_1 = 0.033, c_2 = -0.019, c_3 = -0.010$$

- Note:
  - Wind variance goes down in summer, temperature goes up
  - High in spring and autumn, where it is low for temperature
  - Temperature high variance in winter

Relation to CAR(4)-model $X_1(t)$

- Using Euler approximation on dynamics of $X_1(t)$

$$\begin{aligned} X_1(t) \approx\ & (4-\alpha_1)X_1(t-1) + (3\alpha_1 - \alpha_2 - 6)X_1(t-2) \\ & + (4 + 2\alpha_2 - \alpha_3 - 3\alpha_1)X_1(t-3) \\ & + (\alpha_3 - \alpha_4 - \alpha_2 + \alpha_1 - 1)X_1(t-4) \end{aligned}$$

- Knowing the β's yield

$$\alpha_1 = 3.645, \alpha_2 = 5.039, \alpha_3 = 3.133, \alpha_4 = 0.712$$

- Eigenvalues of A have negative real part, thus stationary dynamics

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