MAT4770/9770
Stochastic Modelling in Energy and Commodity Markets

ASSIGNMENT

Evaluation: passed/not passed
To receive a passing grade all points in the exercises have to be presented with a total of at least 60% correct material. In the evaluation, weight is also given to the presentation.

Delivery of the assignment
You can choose between scanning handwritten notes or typing the solution directly on a computer with a typesetting software for mathematics. It is expected that you give a clear presentation with all necessary explanations. The assignment must be submitted as a single PDF file. It is your responsibility to guarantee a good readable pdf file.

Students who fail the assignment, but have made a genuine effort at solving the exercises, are given a second attempt where they can revise their answers. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, you may be asked to give an oral account.

Application for postponed delivery
If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo [at] math.uio.no) before the deadline. Note that teaching staff can not grant extensions.

The assignment in this course is mandatory to be allowed to take the final examination. Complete guidelines about mandatory assignments are found at the webpage of the course.

Exercise 1
Consider a model for temperatures associated to the CAR(2) process $X(t)$, $t \geq 0$, with parameters: $\alpha_1 = \frac{1}{2}$, $\alpha_2 = \frac{1}{128}$, $\sigma = 2$, and a seasonal component $\Lambda(t) = 184$, for all t:

$$T(t) = \Lambda(t) + X_1(t), \quad t \geq 0.$$

Consider a market with numéraire linked to an instantaneous interest $r = 1/10$ continuously compounded.

1. Write explicitly the equation for X and its solution.
2. Write the Girsanov transform for which X becomes a martingale under the new equivalent probability measure Q. This will be used as pricing measure.
3. Write explicitly the dynamics of X under Q.

4. Write the corresponding time series AR(2) corresponding to the CAR(2) model given.

5. For this model of temperatures, write the model for the CAT index in continuous time with delivery in the period $\tau_1 = 1, \tau_2 = 2$.

6. Derive the future price $F_{CAT}(t, 1, 2)$ at $t < 1$ on the CAT[1,2] using the pricing measure Q above.

7. Derive the price at $t = 0$ of a put option written on $F_{CAT}(t, 1, 2)$, using the pricing measure Q above.

8. Derive the price of the future $F_{CDD}(t, 1, 2)$ on the CDD index in continuous time for delivery in the period $[1, 2]$. Describe how to evaluate $F_{CDD}(t, 1, 2)$ at $t = 0$.

Exercise 2

Consider a model for the Elspot of geometric type

$$S(t) = \exp\{\Lambda(t) + X(t)\}, \qquad t \geq 0,$$

where $\Lambda(t) = \kappa$ and X corresponds to a mean-reverting Ornstein-Uhlenbeck process Brownian driven with positive constant coefficients:

$$dX(t) = (\mu - \alpha X(t))dt + \sigma dB(t), \qquad X(0) = x_0 > 0.$$

Consider an electricity market with numéraire linked to an instantaneous interest r continuously compounded.

1. Find the future price $f(t, \tau)$ for an instantaneous delivery and settlement at τ via a pricing measure Q (use Girsanov transform).

2. Write the stochastic differential equation for this forward price. Use the Itô formula for this.

3. Consider a two (independent) factor geometric model

$$S(t) = \exp\{\Lambda(t) + X(t) + Y(t)\}, \qquad t \geq 0,$$

where $\Lambda(t) = \kappa$ and X corresponds to a mean-reverting Ornstein-Uhlenbeck process Brownian driven with positive constant coefficients:

$$dX(t) = (\mu - \alpha X(t))dt + \sigma dB(t), \qquad X(0) = x_0 > 0.$$

and Y corresponds to a mean-reverting Ornstein-Uhlenbeck process driven by a Compound Poisson process with intensity λ and jump size distribution with second moment (the coefficients β and η are positive constants):

$$dY(t) = (-\beta Y(t))dt + \eta dI(t), \qquad Y(0) = y_0 > 0.$$

Find the future price $f(t, \tau)$ for an instantaneous delivery and settlement at τ via a pricing measure Q (use the Girsanov and the Esscher transform for the two components).