

Lecture 4. Exercise 9

Justify equation (11) pag. 12 in Chigensky's notes.

The model is

$$\begin{cases} dX_t = dW_t \\ dY_t = X_t dt + dV_t \end{cases}$$

V and W two indep. standard Brownian motions.

The Kalman-Bucy filter with

$$a \equiv 0, b \equiv 1, A \equiv 1, B \equiv 1.$$

gives

$$d\hat{X}_t = P_t (dY_t - \hat{X}_t dt)$$

$$\dot{P}_t = 1 - P_t^2$$

$$\Rightarrow \hat{X}_t = \int_0^t P_s (dY_s - \hat{X}_s ds)$$

$$\dot{P} = 1 - P^2$$

→ In the notes is

$$2 - P_t^2,$$

but I think it is a typo.