

1 Linear filtering in discrete time

- Linear projection of one r.v. onto another one.
- Statement of Theorem 1.1 (Orthogonal projection)
- Statement and proof (case that the covariance matrix is non-singular) of Proposition 1.2.
- Recursive orthogonal projection: statement and proof (only sketch) of Proposition 1.4 .
- Setup and statement of the Kalman-Bucy filter.

2 Nonlinear filtering in discrete time

- Review of conditional expectation: know the main properties of conditional expectation.
- Statement and proof of Bayes formula and its version when the conditioning σ -algebra is generated by a random variable. Theorem 2.14 and Corollary 2.15.
- Setup, statement and proof of the filtering recursion, Theorem 2.19.
- Change of measure method setup and proof.
- The Kalman-Bucy when the noises and initial conditions are Gaussian. Statement of Lemma 2.23. Proof of only point (3) in that lemma.

3 Review on stochastic analysis

In the exam the basic results will not be asked explicitly, but you need to know them for solving the problems. You need to know how to use Itô's formula, Girsanov's theorem, martingale representation theorem, integration by parts (or product formula) and Lévy's characterization theorem.

4 Linear filtering in continuous time

- One dimensional case: statement of the setup and main result Theorem 4.1 (No proof)
- General case: statement of the setup and main result Theorem 4.4.

5 Nonlinear filtering in continuous time

- Statement of the main setup: Including the signal as a solution of a martingale problem, specification of the observation process, statement of the problem Definition 5.2 and main general result Theorem 5.4
- Innovation processes: Definition 5.6 and Proposition 5.7 with proof.
- FKK martingale representation. Theorem 5.9 with proof.
- The change of probability measure method. Explain the change of measure, Lemma 5.15 (no proof), reasoning and sketch of proof of Proposition 5.17.
- Kallianpur-Striebel formula: Proposition 5.19 statement and proof. This includes the definition of the unnormalized distribution and Corollary 5.23.
- The Zakai equation statement and sketch of proof. Definition 5.20, Lemma 5.21 and Corollary 5.23 without proofs.
- The Kushner-Stratonovich equation statement and sketch of proof.