MANDATORY ASSIGNMENT MAT4800 FALL 2012. DEADLINE: NOV. 21.

Problem 0.1. (Exercise 2.1 in Forster) Let $\Gamma \subset \mathbb{C}$ be a lattice. The Weierstrass p-function is defined as

$$p_\Gamma(z) = \sum_{\omega \in \Gamma \setminus \{0\}} \frac{1}{(z-\omega)^2} - \frac{1}{\omega^2}.$$

(a) Prove that p_Γ is a well defined meromorphic function on the torus $\pi : \mathbb{C} \to \mathbb{C}/\Gamma$.

(b) Prove that if $f \in \mathcal{M}(\mathbb{C}/\Gamma) \cap \mathcal{O}((\mathbb{C}/\Gamma) \setminus [0])$ has a pole of order 2 at $[0]$, has an expansion

$$f \circ \pi(z) = \sum_{j=-2}^{\infty} c_j z^j, c_{-2} = 1, c_{-1} = c_0 = 0,$$

then $f = p_\Gamma$.

Problem 0.2. Let $\Gamma \subset \mathbb{C}$ be a lattice. Show that the form dz on $\mathbb{C}$ defines a holomorphic 1-form on $\mathbb{C}/\Gamma$.

Problem 0.3. Show that there does not exist any holomorphic 1-form on $\mathbb{P}^1$.

Problem 0.4. Prove the stronger version of Problem 0.1. (b): if $f \in \mathcal{M}(\mathbb{C}/\Gamma) \cap \mathcal{O}((\mathbb{C}/\Gamma) \setminus [0])$ has a pole of order 2 at $[0]$, has an expansion

$$f \circ \pi(z) = \sum_{j=-2}^{\infty} c_j z^j, c_{-2} = 1, c_0 = 0,$$

then $f = p_\Gamma$.

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