MAT4740/9740
Malliavin calculus and applications to finance

Exam.

The exam is held at the blackboard for a duration of about 30-40 minutes in all.

A selection of topics from the list here below will be asked.
The candidate can organize his/her reply bearing in mind:
- to center the topic and address the major results
- to be prepared on the proofs of the statements, as presented/discussed in class or left as exercise after discussion
- to be prepared on the notions that are embedded in the topic addressed.

In addition, the candidate can be asked to use the notions and techniques acquired in some specific context., e.g. short exercise.

Hereafter is a list of topics which are going to be tested during the exam.

PART I - Brownian motion

1. Itô integration and construction of iterated Itô integrals, multiple Itô integrals, properties, connection with Hermite polynomials. Wiener chaos decomposition (a.k.a. Itô chaos expansion).
2. Malliavin derivative: construction from smooth random variables. Representation of the Malliavin operator via chaos expansions.
3. Closedness of linear operators. Malliavin derivative operator on smooth random variables is closable.
4. Skorohod integral and chaos expansions representation, dual operator of the Malliavin derivative, duality and integration by parts, fundamental theorem of calculus. Relationship between Skorohod integral and Itô integral.
5. Itô represenation theorem and the Clark-Haussmann-Ocone formula.
6. Chain rule, Clark-Haussmann-Ocone formula under change of measure, application to hedging in a complete market driven by Brownian motion.
7. Application to sensitivity analysis: computation of the Delta. Specific application of Malliavin methods.

PART II - Centered Poisson random measure

8. Lévy processes, infinitely divisible distribution and the Lévy-Khintchine formula, Lévy measure and Poisson random measure associated to a Lévy process.
9. Lévy-Itô processes, Itô formula for Lévy-Itô processes, Lévy-Itô decomposition theorem; Itô representation theorem with respect to the Poisson random measure.
10. Combination of the Brownian and Poisson structures: product spaces; mixtures of Gaussian and Poisson random measures.
11. Iterated Itô integrals and chaos expansions with respect to the centered Poisson random measure.

12. Malliavin derivative and Skorohod integral with respect to the centered Poisson random measure, the operators, duality, integration by parts, fundamental theorem of calculus. Relationship between Skorohod integral and Itô integral.
13. Chain rule for the Malliavin Derivative with respect to the Poisson random measure.
14. Itô Integral representation theorem and Clark-Haussmann-Ocone formula with respect to the centered Poisson random measure. Combination with the Itô integral representation with respect to the mixture of Gaussian and centered Poisson random measure, Clark-Haussmann-Ocone formula in this last case. Application to minimal variance hedging in incomplete markets driven by Gaussian and centred Poisson noises.
15. Computation of the sensitivity parameters to model risk in dynamics driven by Gaussian and centered Poisson random measure: focus on the Delta. Approaches to the computation: the density method, the conditional density method, the Malliavin method.
16. Existence of density of probability laws and their properties: the problem, the use of integration by parts and duality, the use of Malliaivn caluculus in this context (Brownian motion set-up only).