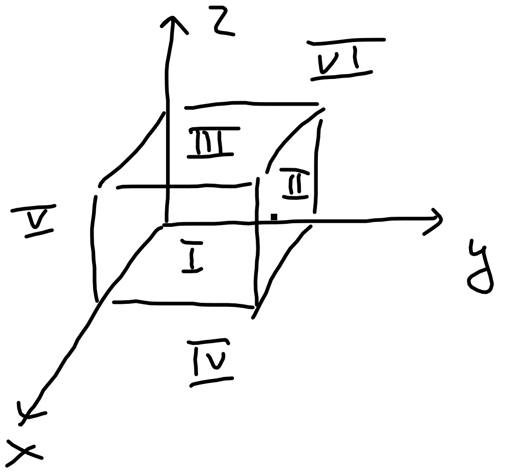
Eksamen V 2017

1a)



$$\int_S \underline{v}\cdot\underline{n}\,d\sigma = \int_I \underline{v}\cdot\underline{n}\,d\sigma + \dots$$

$$\int_I \underline{v}\cdot\underset{\underline{i}}{\underset{\parallel}{\underline{n}}}\,\underset{dy\,dz}{\underset{\parallel}{d\sigma}} = \iint 4x^2y\Big|_{x=1} dy\,dz = \int_0^1\int_0^1 4y\,dy\,dz$$

$$= 2y^2\Big|_0^1 z\Big|_0^1 = 2$$

$$\int_{\underline{II}} \underline{v}\cdot\underline{n}\,d\sigma = \iint xyz\Big|_{y=1} dx\,dz = \int_0^1\int_0^1 xz\,dx\,dz$$

$\underline{n} = \underline{j}$, $\quad d\sigma = dx\,dz$

$$= \frac{1}{2}x^2\Big|_0^1 \; \frac{1}{2}z^2\Big|_0^1 = \frac{1}{4}$$

$$\int_{\underline{III}} \underline{v}\cdot\underline{n}\,d\sigma = \iint yz^2\Big|_{z=1} dx\,dy = \int_0^1\int_0^1 y\,dx\,dy$$

$\underline{n} = \underline{k}$, $\quad d\sigma = dx\,dy$

$$= x\Big|_0^1 \; \frac{1}{2}y^2\Big|_0^1 = \frac{1}{2}$$

$$\int_{\underline{IV}} = \int_{\underline{V}} = \int_{\underline{VI}} = 0$$

$$\int_S \underline{v}\cdot\underline{n}\,d\sigma = 2+\frac{1}{4}+\frac{1}{2} = \frac{11}{4}$$

1b) Gauss sats.

$$\int_S \underline{v}\cdot\underline{n}\, d\sigma = \int_V \nabla\cdot\underline{v}\, d\tau$$

$$\nabla\cdot\underline{v} = \frac{\partial}{\partial x}\left(4x^2y\right) + \frac{\partial}{\partial y}\left(xyz\right) + \frac{\partial}{\partial z}\left(yz^2\right)$$

$$= 8xy + xz + 2yz$$

$$\int \nabla\cdot\underline{v}\, d\tau = \int_0^1\int_0^1\int_0^1 8xy + xz + 2yz \; dx\,dy\,dz$$

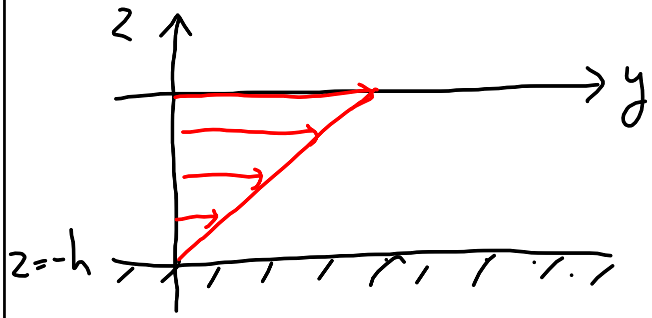
$$= 8\,\tfrac{1}{2}x^2\Big|_0^1\,\tfrac{1}{2}y^2\Big|_0^1\, z\Big|_0^1 + \tfrac{1}{2}x^2\Big|_0^1\, y\Big|_0^1\,\tfrac{1}{2}z^2\Big|_0^1 + 2x\Big|_0^1\,\tfrac{1}{2}y^2\Big|_0^1\,\tfrac{1}{2}z^2\Big|_0^1$$

$$= 2 + \frac{1}{4} + \frac{1}{2} = \frac{11}{4}$$

mai 24-10:32

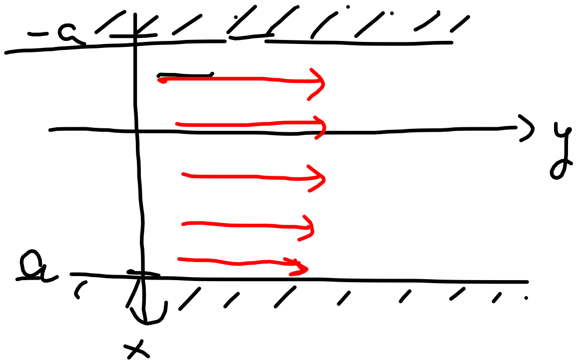
2)

sett fra siden



Skjærstrøm

ovenfra



2a) dersom potensialstrøm

$$\underline{v} = \nabla \phi$$

$$\frac{\partial \phi}{\partial x} = 0 \quad \Rightarrow \phi = f_1(y,z)$$

$$\frac{\partial \phi}{\partial y} = \frac{U(z+h)}{h} \quad \Rightarrow \phi = \frac{U(z+h)y}{h} + f_2(x,z)$$

$$\frac{\partial \phi}{\partial z} = 0 \quad \Rightarrow \phi = f_3(x,y)$$

$$\nabla \times \underline{v} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & \frac{U(z+h)}{h} & 0 \end{vmatrix} = -\underline{i}\,\frac{U}{h} \neq \underline{0}$$

$\nabla \times \underline{v} \neq \underline{0} \iff \underline{v}$ er ikke et potensialfelt

2b) $y = 0$ på tvers av elva

$$\int_{y=0} \underline{v} \cdot \underbrace{\underline{n}}_{=\underline{j}}\, \underbrace{d\sigma}_{=dx\,dz} = \int_{-h}^{0}\int_{-a}^{a} \frac{U(z+h)}{h}\, dx\, dz \qquad \frac{m^3}{s}$$

$$= 2a\,\frac{U}{h}\left[\frac{1}{2}z^2 + hz\right]_{-h}^{0} = \frac{2aU}{h}\left(-\frac{1}{2}h^2 + h^2\right)$$

$$= +\frac{2aU}{h}\,\frac{1}{2}h^2 = Uah$$

mai 24-10:43

2c)

$$\underbrace{\cancel{\frac{\partial \rho}{\partial t}}}_{\to 0} + \underline{v} \cdot \underbrace{\cancel{\nabla \rho}}_{\to 0} + \rho \nabla \cdot \underline{v} = 0$$

$$\nabla \cdot \underline{v} = \frac{\partial}{\partial y}\left(\frac{U(z+h)}{h}\right) = 0$$

2d)

$$\underbrace{\cancel{\frac{\partial \underline{v}}{\partial t}}}_{\to 0} + \underline{v} \cdot \nabla \underline{v} = -\frac{1}{\rho}\nabla p - g\underline{k}$$

Stasjonært hastighetsfelt

$$\underline{v} \cdot \nabla = \frac{U(z+h)}{h}\underline{j} \cdot \left(\underline{i}\frac{\partial}{\partial x} + \underline{j}\frac{\partial}{\partial y} + \underline{k}\frac{\partial}{\partial z}\right)$$

$$= \frac{U(z+h)}{h}\frac{\partial}{\partial y}$$

$$(\underline{v} \cdot \nabla)\underline{v} = \frac{U(z+h)}{h}\frac{\partial}{\partial y}\left(\frac{U(z+h)}{h}\underline{j}\right) = \underline{0}$$

2d) forts. Eulers likning

$$\underline{0} + \underline{0} = -\frac{1}{\rho}\nabla p - g\,\underline{k}$$

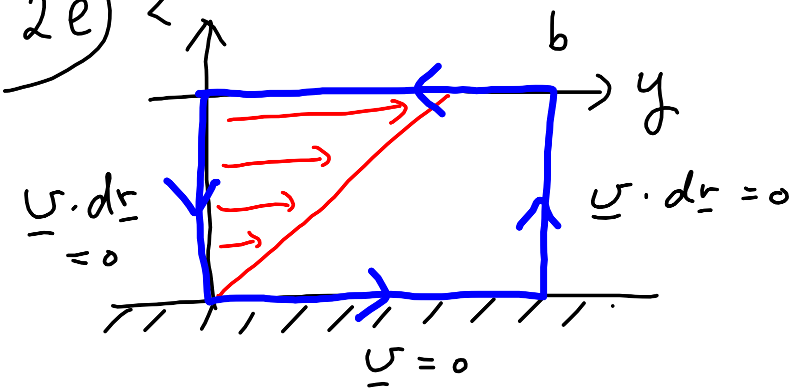
$$\nabla p = -\rho g\,\underline{k}$$

$$\left.\begin{aligned} \frac{\partial p}{\partial x} &= 0 \\ \frac{\partial p}{\partial y} &= 0 \end{aligned}\right\} \Rightarrow p = p(z)$$

$$\frac{\partial p}{\partial z} = -\rho g \Rightarrow p = -\rho g z + f(x,y)$$

$$= -\rho g z + p_0$$

2e)



$$\oint \underline{v}\cdot d\underline{r} = \int_{\text{toppkurve}} \underline{v}\cdot d\underline{r}$$

parameterisering : $\underline{r}(s) = (b-s)\,\underline{j}$ for $0 \le s \le b$

$$d\underline{r} = \frac{d\underline{r}}{ds}\,ds = -\underline{j}\,ds$$

$$\int_0^b +U\underline{j}\cdot(-\underline{j}\,ds) = \int_0^b -U\,ds = -Ub$$

2e forts.) Stokes sats $\equiv$ Stokes teorem

$$\oint_\gamma \underline{v}\cdot d\underline{r} = \int_S \underbrace{\nabla\times\underline{v}}_{= -\underline{i}\frac{U}{h}} \cdot \underbrace{\underline{n}}_{=\underline{i}}\, \underbrace{d\sigma}_{= dy\,dz}$$

$$= \int_{-h}^{0}\int_{0}^{b} -\frac{U}{h}\,dy\,dz = -\frac{Uhb}{h} = -Ub$$

2d Kommentar

Kunne vi ha brukt Bernoullis likning?

$$\frac{p}{\rho} + \frac{v^2}{2} + gz = \text{konstant langs en strømlinje}$$

Kan ikke brukes fordi $\nabla\times\underline{v} \neq 0$

mai 24-11:06

3) $\underline{v} = -v_0\, \underline{i}_r + \omega r\, \underline{i}_\theta$

3a) $\nabla \cdot \underline{v} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\, v_r\right) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$

$= \frac{1}{r}\frac{\partial}{\partial r}\left(-r v_0\right) + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\omega r\right)$

$= -\frac{v_0}{r} < 0$

en kontraksjon

3b) $\nabla \times \underline{v} = \frac{1}{r}\begin{vmatrix} \underline{i}_r & r\underline{i}_\theta & \underline{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ -v_0 & r\omega r & 0 \end{vmatrix} = \frac{\underline{k}}{r}\frac{\partial}{\partial r}\left(\omega r^2\right) = 2\omega\underline{k} = 2\underline{\omega}$

3c) $$\underline{a} = \frac{D\underline{v}}{dt} = \underbrace{\cancel{\frac{\partial \underline{v}}{\partial t}}}_{\to 0} + \underline{v}\cdot\nabla\underline{v}$$

$$\underline{v} = -v_0\,\underline{i}_r + \omega r\,\underline{i}_\theta$$

$$\underline{v}\cdot\nabla = \left(-v_0\underline{i}_r + \omega r\,\underline{i}_\theta\right)\cdot\left(\underline{i}_r\frac{\partial}{\partial r} + \frac{\underline{i}_\theta}{r}\frac{\partial}{\partial\theta} + \underline{k}\frac{\partial}{\partial z}\right)$$

$$= -v_0\frac{\partial}{\partial r} + \omega\frac{\partial}{\partial\theta}$$

$$\left(\underline{v}\cdot\nabla\right)\underline{v} = \left(-v_0\frac{\partial}{\partial r} + \omega\frac{\partial}{\partial\theta}\right)\left(-v_0\underline{i}_r + \omega r\,\underline{i}_\theta\right)$$

$$= -\omega v_0\,\underline{i}_\theta - \omega v_0\frac{\partial \underline{i}_r}{\partial\theta} + \omega^2 r\frac{\partial \underline{i}_\theta}{\partial\theta}$$

Husk $\underline{i}_r = \underline{i}\cos\theta + \underline{j}\sin\theta$

$\underline{i}_\theta = -\underline{i}\sin\theta + \underline{j}\cos\theta$

$$\frac{\partial \underline{i}_r}{\partial\theta} = -\underline{i}\sin\theta + \underline{j}\cos\theta = \underline{i}_\theta$$

$$\frac{\partial \underline{i}_\theta}{\partial\theta} = -\underline{i}\cos\theta - \underline{j}\sin\theta = -\underline{i}_r$$

$$\underline{a} = -\omega v_0 \underline{i}_\theta - \omega v_0 \underline{i}_\theta - \omega^2 r \underline{i}_r$$

$$= \underbrace{-2\omega v_0 \underline{i}_\theta}_{\text{Coriolis akselerasjon}} \underbrace{- \omega^2 r \underline{i}_r}_{\text{Sentripetal akselerasjon}}$$

I foajéen til Fysisk institutt
er det en Foucault pendel