1a) $\nabla \cdot \underline{v} = \frac{\partial}{\partial x} x + \frac{\partial}{\partial y}(-y) = 1 - 1 = 0$

$$\nabla \times \underline{v} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & -y & 0 \end{vmatrix} = \underline{0}$$

1b) $\nabla \times \underline{v} = \underline{0} \Rightarrow$ Det eksisterer et potensial ϕ

$\underline{v} = \nabla \phi$

$$\left.\begin{aligned} \frac{\partial \phi}{\partial x} &= x \Rightarrow \phi = \frac{1}{2}x^2 + f(y) \\ \frac{\partial \phi}{\partial y} &= -y \Rightarrow \phi = -\frac{1}{2}y^2 + g(x) \end{aligned}\right\} \Rightarrow \phi = \frac{1}{2}x^2 - \frac{1}{2}y^2 + C$$

$\nabla \cdot \underline{v} = 0$ og $\underline{v}$ er 2D $\Rightarrow$ Det eksisterer en strømfunksjon ψ

$$\underline{v} = \nabla \times \underline{A} = \nabla \times (-\underline{k}\,\psi) = \underline{k} \times \nabla \psi$$

$$\left.\begin{aligned} v_x &= -\frac{\partial \psi}{\partial y} = x \Rightarrow \psi = -xy + f(x) \\ v_y &= \frac{\partial \psi}{\partial x} = -y \Rightarrow \psi = -xy + g(y) \end{aligned}\right\} \Rightarrow \psi = -xy + B$$

1c) Stagnasjonspunkt $\quad \underline{v} = x\,\underline{i} - y\,\underline{j} = \underline{0}$

$x = y = 0 \quad$ Origo

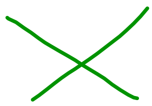
Diskriminant regel: $\quad Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$B^2 - 4AC$ > 0 hyperbler
$= 0$ parabler
< 0 ellipser, sirkler

$\frac{1}{2}x^2 - \frac{1}{2}y^2 \quad$ diskriminant $= 1 > 0 \quad$ hyperbler

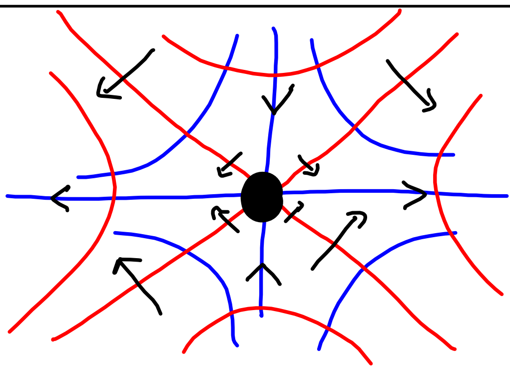
$-xy \quad$ diskriminant $= 1 > 0 \quad$ — " —

$\frac{1}{2}x^2 - \frac{1}{2}y^2 = \frac{1}{2}(x-y)(x+y) = 0$



$\psi = -xy = \text{konstant}$

$\phi = \frac{1}{2}x^2 - \frac{1}{2}y^2 = \text{konstant}$



$\underline{v} = x\underline{i} - y\underline{j}$

$|\underline{v}| = \sqrt{x^2+y^2}$

1d) Parameterisering $\underline{r}(\theta) = \underline{i}\cos\theta + \underline{j}\sin\theta$

Infinitesimalt kurveelement $d\underline{r} = \frac{d\underline{r}}{d\theta}\,d\theta = (-\underline{i}\sin\theta + \underline{j}\cos\theta)\,d\theta$

$\underline{v} = x\underline{i} - y\underline{j} = \cos\theta\,\underline{i} - \sin\theta\,\underline{j}$

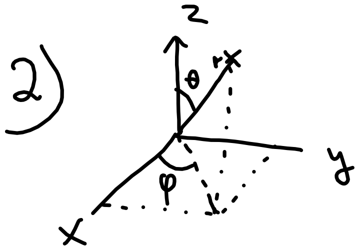
$\underline{v}\cdot d\underline{r} = (-\sin\theta\cos\theta - \sin\theta\cos\theta)\,d\theta = -2\sin\theta\cos\theta\,d\theta$

$= -\sin 2\theta\,d\theta$

$$\oint \underline{v}\cdot d\underline{r} = \int_0^{2\pi} -\sin 2\theta\,d\theta = \frac{1}{2}\cos 2\theta\Big|_0^{2\pi} = \frac{1}{2}(1-1) = 0$$

Stokes sats $\oint \underline{v} \cdot d\underline{r} = \int_S \underbrace{\nabla \times \underline{v}}_{=0} \cdot \underline{n}\, d\sigma = 0$

2)



2a) $\alpha \sim m^{-2}$ $\quad K \sim \frac{m^2}{s}$

$T_0 \sim K$ $\quad \underline{H} \sim \frac{J}{s\, m^2} = \frac{W}{m^2}$

$k \sim \frac{J}{s\, m\, K}$

$$T = T_0 e^{-\alpha r^2} + T_1$$

$$\nabla T = \underline{i}_r \frac{\partial T}{\partial r} = -\underline{i}_r T_0 \alpha\, 2r\, e^{-\alpha r^2}$$

$$\underline{H} = -k \nabla T = 2\alpha k T_0 r\, \underline{i}_r\, e^{-\alpha r^2}$$

$\underline{n} = \underline{i}_r$

$$\int_{r=R} \underline{H} \cdot \underline{n}\, d\sigma = \int_{r=R} 2\alpha k T_0 R\, e^{-\alpha R^2} d\sigma$$

$$= 2\alpha k T_0 R e^{-\alpha R^2} \int d\sigma = 4\pi R^2$$

$$\int d\sigma = \int_0^{2\pi}\int_0^{\pi} r^2 \Big|_{r=R} \sin\theta \, d\theta \, d\varphi = R^2 \, 2\pi \left[-\cos\theta\right]_0^{\pi} = 4\pi R^2$$

2c) $$\frac{\partial T}{\partial t} = \kappa \nabla^2 T$$

$$\nabla T = \underline{i}_r \frac{\partial T}{\partial r}$$

$$\nabla^2 T = \nabla \cdot \nabla T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right)$$

$$= \kappa \frac{1}{r^2}\frac{\partial}{\partial r}\left(-r^2 \, 2\alpha r T_0 \, e^{-\alpha r^2}\right)$$

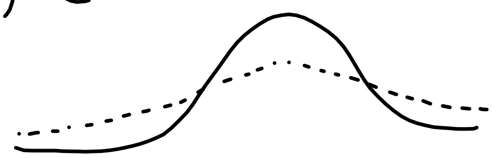
$$= \frac{\kappa}{r^2}\frac{\partial}{\partial r}\left(-2\alpha T_0 \, r^3 \, e^{-\alpha r^2}\right)$$

$$= -\frac{2\alpha \kappa T_0}{r^2}\left(3r^2 - 2\alpha r^4\right) e^{-\alpha r^2}$$

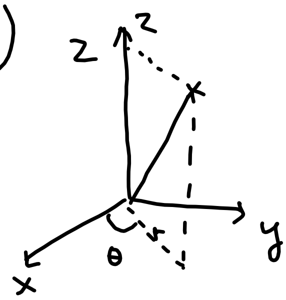
$$\frac{\partial T}{\partial t} = -2\alpha\kappa T_0 \left(3 - 2\alpha r^2\right) e^{-\alpha r^2}$$

$r > \sqrt{\frac{3}{2\alpha}}$ T øker

$r < \sqrt{\frac{3}{2\alpha}}$ T avta



3)



$$\underline{v} = c\sqrt{r}\,\underline{i}_\theta$$

3a) $$\nabla \cdot \underline{v} = \frac{1}{r}\frac{\partial}{\partial \theta} v_\theta = \frac{1}{r}\frac{\partial}{\partial \theta}\left(c\sqrt{r}\right) = 0$$

$$\nabla \times \underline{v} = \frac{1}{r}\begin{vmatrix} \underline{i}_r & r\,\underline{i}_\theta & \underline{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ 0 & r c\sqrt{r} & 0 \end{vmatrix} = \frac{1}{r}\underline{k}\,\frac{\partial}{\partial r}\left(c\, r^{\frac{3}{2}}\right) = \frac{1}{r}\underline{k}\,\frac{3}{2} c\, r^{\frac{1}{2}}$$
$$= \frac{3}{2} c \frac{1}{\sqrt{r}}\,\underline{k} \neq 0$$

3b) $\nabla \cdot \underline{v} = 0$ og $\underline{v}$ = 2D $\Rightarrow$ ψ eksisterer

$$\underline{v} \times d\underline{r} = \underline{0}$$

$$= \begin{vmatrix} \underline{i}_r & \underline{i}_\theta & \underline{k} \\ 0 & c\sqrt{r} & 0 \\ dr & r\,d\theta & dz \end{vmatrix} = \underline{i}_r \underbrace{c\sqrt{r}\,dz}_{=0} + - \underbrace{c\sqrt{r}\,dr}_{=0}\,\underline{k} = \underline{0}$$

z = konstant r = konstant

Strømlinjene er sirkler rundt z-aksen.

3b) $$\underline{a} = \underbrace{\cancel{\frac{\partial \underline{v}}{\partial t}}}_{\to 0} + \underline{v} \cdot \nabla \underline{v}$$

$$\underline{v} \cdot \nabla = c\sqrt{r}\, \underline{i}_\theta \cdot \left(\underline{i}_r \frac{\partial}{\partial r} + \frac{1}{r} \underline{i}_\theta \frac{\partial}{\partial \theta} + \underline{k} \frac{\partial}{\partial z} \right) = \frac{c}{\sqrt{r}} \frac{\partial}{\partial \theta}$$

$$(\underline{v} \cdot \nabla)\, \underline{v} = \frac{c}{\sqrt{r}} \frac{\partial}{\partial \theta} \left(c\sqrt{r}\, \underline{i}_\theta \right) = c^2 \frac{\partial \underline{i}_\theta}{\partial \theta} = -c^2 \underline{i}_r$$

$$\underline{i}_\theta = -\sin\theta\, \underline{i} + \cos\theta\, \underline{j} \qquad \frac{\partial \underline{i}_\theta}{\partial \theta} = -\cos\theta\, \underline{i} - \sin\theta\, \underline{j} = -\underline{i}_r$$

$$\underline{a} = -c^2 \underline{i}_r$$

3c) Eulers likning $$\underbrace{\cancel{\frac{\partial \underline{v}}{\partial t}}}_{\to 0} + \underbrace{\underline{v} \cdot \nabla \underline{v}}_{-c^2 \underline{i}_r} = -\frac{1}{\rho} \nabla p - g \underline{k}$$

$$\frac{D\underline{v}}{dt} \equiv \frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}$$

$$\nabla p = \rho c^2 \underline{i}_r - \rho g \underline{k}$$

$$\left.\begin{aligned} \frac{\partial p}{\partial r} &= \rho c^2 \\ \frac{1}{r}\frac{\partial p}{\partial \theta} &= 0 \\ \frac{\partial p}{\partial z} &= -\rho g \end{aligned}\right\} \Rightarrow p = \rho c^2 r - \rho g z + \text{konstant}$$

på $z = \eta(r)$ $\quad p = p_0$

$$p_0 = \rho c^2 r - \rho g \eta(r) + \text{konstant}$$

$$\eta(r) = \frac{c^2 r}{g} + \text{konstant}$$