Volumintegral

M 2,3

Volumintegraler summerer over <u>volumelementer</u>.

Det kreves ingen ny forståelse, utover det vi kan for kurve- og flateintegraler

1D

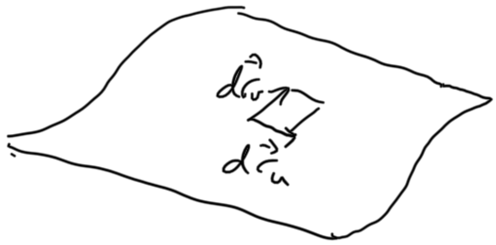
bueelement

lengde $|d\vec{r}|$

Parametrisering i 1 parameter

Kartesisk, langs x-aksen $\rightarrow ds = dx$

2D



Flateelement

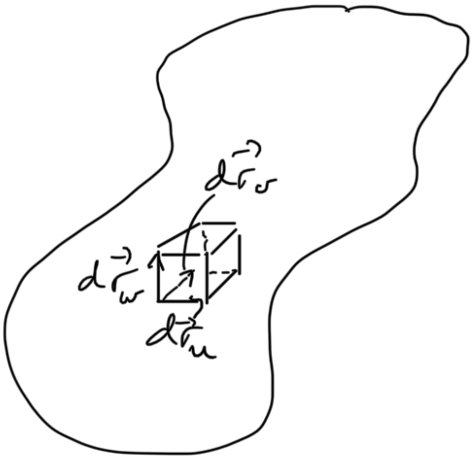
Areal $|d\vec{r}_u \times d\vec{r}_v|$

Parametrisering u, v

Kartesisk i xy-planet $\rightarrow d\sigma = dx\,dy$

Volumelement

3D



Volum $|d\vec{r}_w \cdot d\vec{r}_u \times d\vec{r}_v|$

Skalar triple produkt

Parametrisering i u, v, w

Kartesisk 3D rom $\rightarrow dV = dx\,dy\,dz$

Volumet deles inn i volum-elementer, og summeres

$$\text{Total volum} = \int_\Omega dV = \iiint dx\,dy\,dz$$

$$\int_\Omega dV = \lim_{N\to\infty} \sum_{i=1}^{N} \Delta V_i$$

der ΔV_i er et volumelement.

Kartesisk $dV = dx\,dy\,dz$

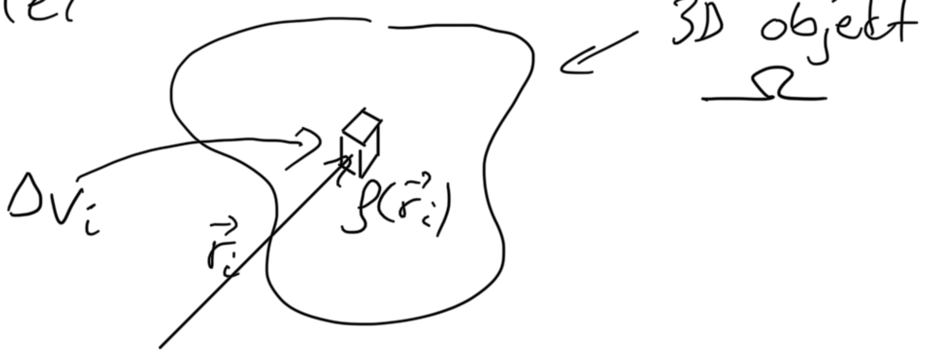
Kurvelineært -> Triple produktet
Kommer tilbake
senere -> kap 6.

Trippelintegraler. Kartesisk:

$$\int_{\Omega} dV = \int_{z_0}^{z_1}\int_{y_0}^{y_1}\int_{x_0}^{x_1} dx\,dy\,dz$$

Total masse av et objekt
med tetthet $\rho(\vec{r})$.

Del objektet inn i N små
deler



$$\text{Totalt volum} = \sum_{i=1}^{N} \Delta V_i$$

Massen til <u>ett</u> volumelement i

$$= \rho(\vec{r}_i)\,\Delta V_i$$

$$\left[\frac{kg}{\cancel{m^3}}\right]\cdot\left[\cancel{m^3}\right]$$

$$\text{Total masse} = \sum_{i=1}^{N} \rho(\vec{r}_i)\,\Delta V_i$$

$$\lim_{N\to\infty} \sum_{i=1}^{N} \rho(\vec{r}_i)\,\Delta V_i = \int_{\Omega} \rho(\vec{r})\,dV$$

I Kartesiske koordinater

$$\int \rho(\vec{r})\,dV = \iiint \rho(x,y,z)\,dx\,dy\,dz$$

<u>Eksempel</u>

En kube $\Omega = [0,1]^3$ har tetthet

$$\rho(x,y,z) = 1 + x + y + z \qquad \left[\frac{kg}{m^3}\right]$$

Finn total masse til kuben.

$$\text{Total masse} = \int_{\Omega} \rho \, dV$$

$$= \int_0^1 \int_0^1 \int_0^1 1 + x + y + z \, dx\,dy\,dz$$

$$= \int_0^1 \int_0^1 \left[(1+y+z)x + \frac{1}{2}x^2 \right]_0^1 dy\,dz$$

$$= \int_0^1 \int_0^1 1 + y + z + \frac{1}{2} \, dy\,dz$$

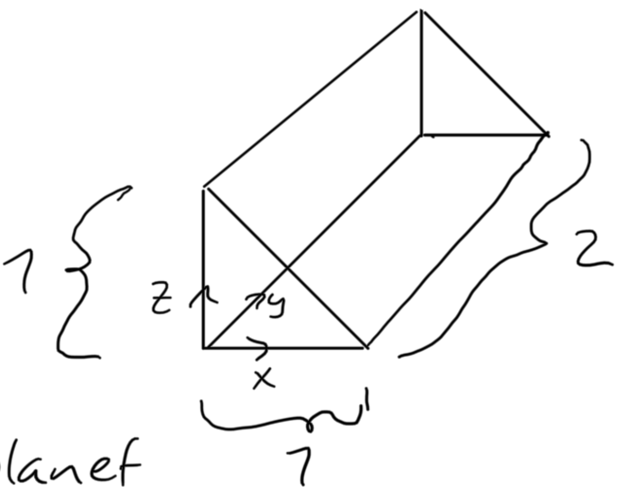
$$= \int_0^1 \left[\left(\frac{3}{2} + z\right) y + \frac{1}{2} y^2 \right]_0^1 dz$$

$$= \int_0^1 2 + z \, dz = \left[2z + \frac{1}{2} z^2 \right]_0^1$$

$$= \underline{\underline{\frac{5}{2}}}$$

Eksempel

Et volum er gitt ved at en rettvinklet trekant i xz-planet trekkes en lengde 2 langs y-aksen. Finn volumet som et integral.



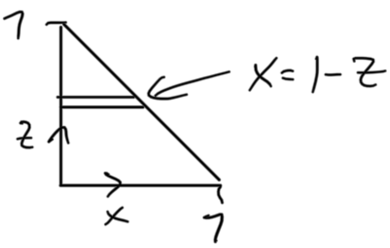
Ikke integral

$$\text{Volum} = \text{Areal trekant} \cdot 2$$

$$= \frac{1}{2} \cdot 2 = \underline{1}$$

Integral

$$\int_0^1 \int_0^2 \int_0^{1-z} dx\,dy\,dz$$



$$\int_0^1 \int_0^2 1-z \,dy\,dz = \int_0^1 (1-z) \int_0^2 dy\,dz$$

$$= 2\int_0^1 (1-z)\,dz = 2\left[z - \frac{1}{2}z^2\right]_0^1$$

$$= 2\left(1 - \frac{1}{2}\right) = \underline{\underline{1}}$$