MEK3200, Project work in mechanics

Lagrangian description of hydrodynamic waves

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There are two classical descriptions for continuum mechanics: the Lagrangian and the Eulerian.

The Lagrangian or material description expresses properties such as displacements of particles with respect to material coordinates and time. This is often the preferred description for solids where particles usually have small displacements.

The Eulerian or spatial description expresses properties such as velocity with respect to fixed spatial coordinates and time. This is often the preferred description for fluids where particles move about with arbitrarily large displacements.

The Lagrangian description is hardly mentioned in introductory courses to fluid mechanics and hydrodynamic waves, e.g. MEK1100, MEK3230, MEK4320.

The Lagrangian description gives rise to some curious solutions for hydrodynamic waves, e.g. the Gerstner exact wave solution which is not based on potential flow theory because it has finite vorticity. While these solutions may have questionable physical properties, since the vorticity they describe is not necessarily anticipated to occur in the ocean, these solutions may be advantageous for certain applications, e.g. in the movie industry where film makers may desire more exciting animations of a liquid surface than the dull animations that typically arise from potential flow theory.

A nice review of this topic, including the topic of movie animations, can be found here: `https://en.wikipedia.org/wiki/Trochoidal_wave`

Goals of this project:

1. Describe how the continuity equation and the conservation of momentum and the surface conditions at the interface between liquid and air should be expressed in Lagrangian coordinates.

2. Describe some wave solutions in the Lagrangian description. In particular describe the Gerstner wave.

3. Compare the properties of the Gerstner wave with the classical linear potential wave theory in Eulerian coordinates.

4. (Optional) Compare the Gerstner wave with the nonlinear Stokes wave.

5. Produce computer animations of a liquid surface with the Gerstner wave in Lagrangian coordinates, and with potential wave theory in Eulerian coordinates. Discuss which of these wave animations would be most suitable to entertain an audience in a movie theater.

6. (Optional) The arbitrary Lagrangian–Eulerian (ALE) description was developed in an attempt to combine the advantages of both the Lagrangian and Eulerian descriptions. While the Lagrangian description is seldom used in fluid mechanics, the ALE description is used for numerical work. Write a review of the ALE description.