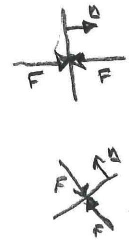
09/12/2015

Hydrostatikk – likevekt i fluider

Spesial tilfelle, statisk fluid $\rightarrow$ ingen bevegelse

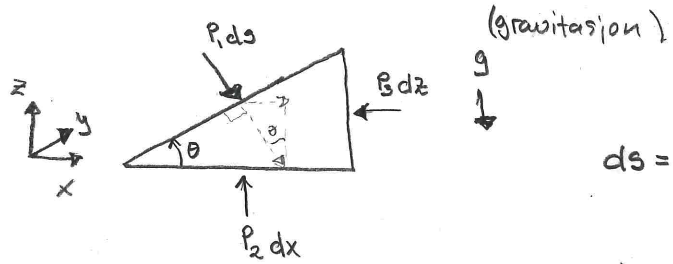
Likevekt = balanse av normal stress

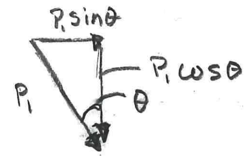
= trykk balanse $[P] = Pa = N/m^2 = \frac{\text{kraft}}{\text{areal}}$



Tangensielle / skjærkrefter $\equiv 0$

Triangulert volum (2D-triangel)





$ds = \frac{dz}{\sin\theta}$

$ds = \frac{dx}{\cos\theta}$

$\Sigma F = 0 \Rightarrow \underline{a} = 0$, ingen akselerasjon

x-retning; $(P_1 ds) \cdot \sin\theta - P_3 dz = 0$ (kraft per lengde)

$P_1 dz = P_3 dz \Rightarrow \underline{P_1 = P_3}$

Gravitasjonen; $\underline{F}_g = -mg\, \underline{e}_z$ $\quad m = \rho \cdot \frac{dx\, dz}{2}$

$\rightarrow$ kraften F_g kan utrykkes som derivatet av et potensial = konservativ kraft

$F_g = -\rho \nabla \chi = -\rho \frac{\partial}{\partial z}(gz)$

$\uparrow$ Potensial eng.

y-retning; $F_g + P_2 dx - (P_1 ds) \cdot \cos\theta = 0$

$-\rho g \frac{dx\, dz}{2} + P_2 dx - P_1 dx = 0$

$\underline{P_2 - P_1 = \frac{\rho g dz}{2}}$ $\left(\Rightarrow \frac{dP}{dz} = -\frac{\rho g}{2} \right)$

Ser på grensen $dx, dy, ds \rightarrow 0$, dvs. trianglet krympes til et enkelt punkt

$\Rightarrow dz \rightarrow 0$; $P_1 = P_2 = P_3$ trykket uavhengig av retning/orientering av overflaten og er en skalar (Pascal's law)

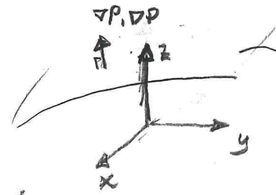
$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial y} = 0$

Trykkfordelingen i et statisk fluid (rektangulært volum/areal)

$$\frac{dP}{dz} = -\rho g \quad \Rightarrow \text{ generell beskrivelse } \quad \underbrace{\frac{1}{\rho}\nabla P = f_v = -\nabla \chi}$$

potensial-flate

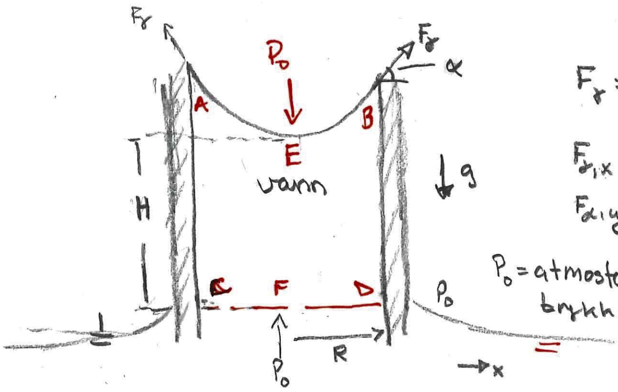
$$\underline{P = P_0 - \rho g z}$$



$\nabla P \times \nabla \chi = \nabla \rho \times \nabla \chi \equiv 0$

P, ρ, χ = ekviskalar flater

Eks. 1) Hvor høy blir vannkolonnen i et tynt rør?



$F_\gamma = \gamma \cdot 2\pi R$

$F_{\gamma,x} = \gamma \cdot 2\pi R \cdot \cos\alpha$

$F_{\gamma,y} = \gamma\, 2\pi R \cdot \sin\alpha$

P_0 = atmosfærisk trykk

$[\gamma] = N/m$ = overflatespennings koeff.

$[\alpha] = [\ ]$ = Kontakt vinkel

$\alpha < 90°$ → vannet liker solidet (hydrophilic)

$\alpha > 90°$ → liker ikke solidet (hydrophobic)

x-retning: $\sum F_x = 0$

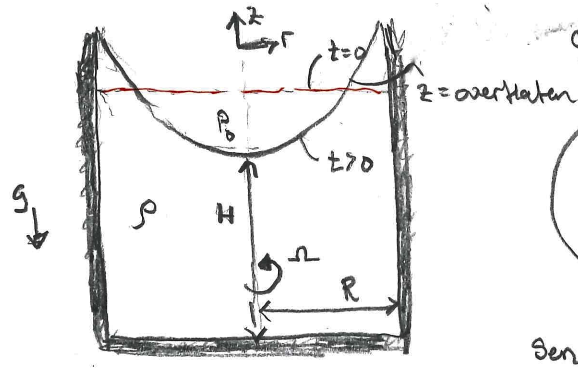
y-retning: $\underbrace{(\cancel{\pi R^2 P_0}) - (\pi R^2 \rho g h)}_{\text{bunnen}} + \underbrace{(\cancel{-\pi R^2 P_0}) + \gamma 2\pi R}_{\text{toppen}} =$

$$\underline{h = \frac{2\gamma \sin\alpha}{\rho g R}}$$

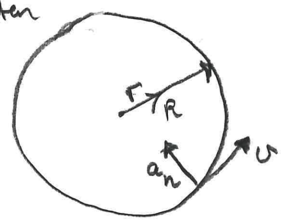
(Jurin's lov, $f(\alpha)$ = overflate kjemi)

Hva skjer med høyden dersom veggen til røret er hydrofobt: $\alpha > 90°$?

Roterende vøtte med væske (Newton's vøtte)



Ovenfra



Ω = vinkel frekvens

$v = \Omega R$

Sentripetal akselerasjon; $a_n = \frac{v^2}{r} = \frac{(r\Omega)^2}{r} = r\Omega^2$

$\underline{a}_n = -\Omega^2 r \frac{\underline{r}}{r} = -\Omega^2 \underline{r}$

$\underline{r} = x\,\underline{e}_x + y\,\underline{e}_y$

$\Rightarrow r^2 = x^2+y^2 = \beta$

$\nabla\beta = 2x\,\underline{e}_x + 2y\,\underline{e}_y = 2\,\underline{r}$

$\Rightarrow \underline{a}_n = -\nabla\left(\frac{\Omega r^2}{2}\right)$

Bevegelses ligningen; $\underline{a} = -\frac{1}{\rho}\nabla p + \underline{g}$

$\Rightarrow a_n = -\Omega^2\underline{r} = -\nabla\left(\frac{\Omega r^2}{2}\right) = -\nabla\left(\frac{p}{\rho} - g\right)$

$$\Rightarrow \nabla\left(\frac{\Omega r^2}{2} + \frac{p}{\rho} - g\right) = 0$$

Antar potensialet, $\phi = -gz + \frac{\Omega r^2}{2}$, $\Rightarrow \nabla(\underbrace{p/\rho + \phi}_{\text{konst.}}) = 0$

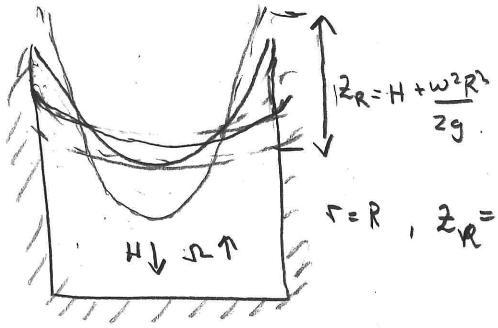
grensebetingelser

$z = H$, $x, y = 0 = r$; $P = P_0 = \rho g H$

(Senter av vøtta)

$$\Rightarrow P = P_0 + \rho g(H-z) + \frac{\rho\Omega^2}{2}(x^2+y^2)$$

Ved overflaten $z = H$, $P = P_0 \Rightarrow -\rho g(H-z) = \frac{\rho\Omega^2}{2}(x^2+y^2)$



$z = H + \frac{\rho\Omega^2}{2g}(x^2+y^2) = \underbrace{H + \frac{\rho(\Omega r)^2}{2g}}_{\text{Paraboloide}}$

$r = R$, $z_R = H + \frac{\rho\Omega^2 R^2}{2g}$

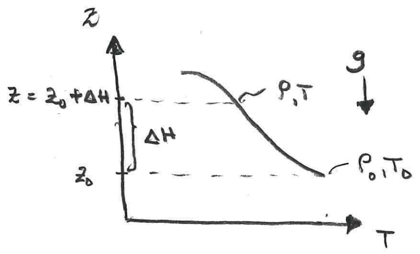
Øker Ω, hva med H?

$\left.\begin{array}{l} \rightarrow p = \text{isobar flate} \\ \phi = \text{nivåflate} \end{array}\right\}$ Sammenfaller

$\nabla p \times \nabla\phi = 0$

vis selv!

Eks. 2, Hydrostatikk i atmosfæren – Ideell gasslov, temp., stabilitet?



Antar atmosfæren kan beskrives som ideell gass

Tilstandsligning, $P = \rho R T$ $[R] = $ gass-konst $= \frac{m^2}{s^2 K}$

$\underline{\rho = \frac{P}{RT}}$ → kobler densitet(ρ), trykk (P) og temperatur (T)

Hydrostatisk trykkligning

$$\frac{dP}{dz} = -\rho g = \frac{-P}{RT} g$$

$$\int_{P_0}^{P} \frac{dP}{P} = \int_{z_0}^{z} -\frac{g}{RT} dz \Rightarrow \underline{P = P_0 \exp\left(-\frac{g}{RT}\int_{z_0}^{z} dz\right) = P_0 \exp\left(-\frac{g}{RT}(z - z_0)\right)}$$

(trykket avtar eksponensielt)

Antar $T(z) \approx T_0$, dvs. konstant temperatur

H_0 = Karrakteristisk høydeskala $= \frac{RT}{g}$. $P = P_0 \exp\left(-\frac{(z - z_0)}{H_0}\right)$ $\wedge\ z_0 = 0$

$$\underline{P = P_0 \exp\left(-\frac{z}{H_0}\right)}$$

$P_0 = 1\,atm = 101\,kPa = 10^5\,Pa = 1\,bar$.

$T = 293\,K$ $(20°C)$, $R = 287 \frac{J}{kgK}$, $g = 9.81\,m/s^2 \Rightarrow H_0 = \frac{290 \cdot 287}{10} m \approx 9 \cdot 10^3 m \approx \underline{9\,km}$

Trykk forskjellen mellom Oslo → $P \sim P_0$, $z_0 = 0$

Galdhøpiggen → $z \approx 2500\,m$ } $\frac{P_{Gald.}}{P_{Oslo}} = \exp\left(-\frac{2500}{9000}\right) = \underline{0.75}$

$\Delta P = P_0 \cdot (1 - 0.75) = \underline{2.5 \cdot 10^4\,Pa}$

(vannsøyle $\approx$ $\Delta P = \rho g h = 10^4 h$ $= \underline{2.5 \cdot 10^4 m}$)

?

Regn selv; hvordan endres beregningen dersom;

i) $T(z) = T_0 - \beta(z - z_0)$ (Barotropisk temp. fordeling)