UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: MEK4100 — Mathematical Methods in Mechanics

Day of examination: Thursday 1. December 2016

Examination hours: 14.30 – 18.30

This problem set consists of 4 pages.

Appendices: Formula sheet

Permitted aids: Mathematical handbook, by K. Rottmann.
Approved calculator

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1 (weight 25%)

We seek periodic, nonlinear solutions of

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + \left(1 + \epsilon \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2\right) y = 0, \quad y(0) = 1, \quad \frac{\mathrm{d}y(0)}{\mathrm{d}t} = 0,$$

where $\epsilon \ll 1$.
Find a periodic approximation correct to (and including) order ϵ by an appropriate technique. Explain your steps.

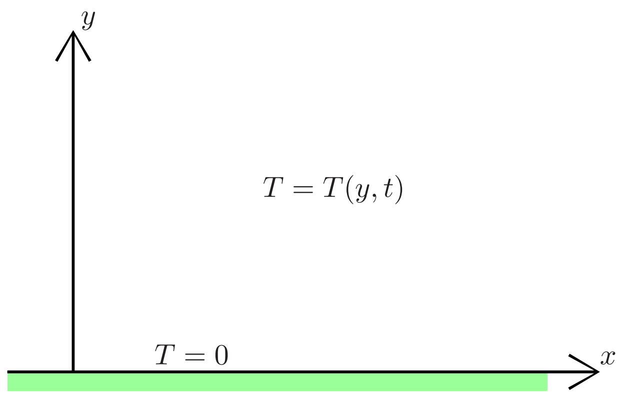
Problem 2 (weight 15%)

From a laboratory experiment (details are not relevant and are thus omitted) the following problem arises

$$g = f\,(x - g)\,,$$

where f is a given function and the function $g(x)$ is the unknown. We now assume that f is small in the sense $f = \epsilon F$, where ϵ is a small and positive number and F is of order 1. Find the two first terms in a perturbation expansion for g.

Problem 3 (weight 20%)



The semi-infinite space above the xy-plane is filled with a material which initially inherits a uniform temperature. For positive times, $t \geq 0$, the material is cooled by applying a different, fixed temperature at the xy plane. Using the temperature in the material minus the temperature at the xy-plane as primary unknown the temperature evolution is governed by the combined boundary and initial value problem (do not show this)

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2}, \quad T(0,t) = 0, \quad T(y,0) = T_0 \tag{1}$$

where T is the initial difference in temperature between material and the xy-plane. Moreover, κ is a constant and T_0 is the initial difference in temperature.

3a (weight 10%)

Find a complete set of non-dimensional number from the quantities involved in (1)

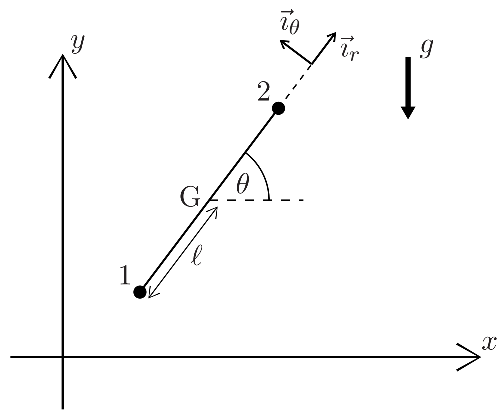
3b (weight 10%)

Show that

$$T = T_0 F(\hat{\pi}),$$

where F is a function of a dimensionless number $\hat{\pi}$. Explain why this suggests that F is determined by an ordinary differential equation, with boundary conditions. Find this boundary value problem, but do not solve it.

Problem 4 (weight 20%)



Two mass particles, denoted by 1 and 2, respectively, move in the vertical x, y plane under the influence of a uniform gravity field. The particles both have mass m and they are connected by a mass-less inflexible rod of length 2ℓ. The center of gravity, $\vec{r}_G = x_G\vec{\imath} + y_G\vec{\jmath}$, is thus located at the mid-point of the rod. Using x_G, y_G and the angle of the rod, θ, as coordinates we obtain (do not show this) the positions and velocities of the two particles on the form (see figure)

$$\vec{r}_1 = \vec{r}_G - \ell\vec{\imath}_r, \quad \vec{r}_2 = \vec{r}_G + \ell\vec{\imath}_r,$$
$$\vec{v}_1 = \vec{v}_G - \ell\dot{\theta}\vec{\imath}_\theta, \quad \vec{v}_2 = \vec{v}_G + \ell\dot{\theta}\vec{\imath}_\theta,$$

where $\vec{v}_G = \dot{x}_G\vec{\imath} + \dot{y}_G\vec{\jmath}$ and $\dot{}$ denotes time differentiation. Moreover, $\vec{\imath}_r$ and $\vec{\imath}_\theta$ are unit vectors parallel and normal to the rod, respectively.

4a (weight 10%)

Find the Lagrangian for this system.

4b (weight 10%)

Find all the first integrals for the Lagrange-equations and explain the physical significance of at least one of them.

Problem 5 (weight 20%)

We are given the differential equation

$$\epsilon^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - W(x)y = 0,$$

where $\epsilon \ll 1$ and $W > 0$ everywhere. We do not bother with boundary conditions in this problem. Use the WKB method to derive that a leading order approximation to the solution of the differential equation is

$$y \approx AW^{-\frac{1}{4}} e^{\frac{1}{\epsilon}\int_{x_a}^{x} W^{\frac{1}{2}} d\hat{x}} + BW^{-\frac{1}{4}} e^{-\frac{1}{\epsilon}\int_{x_a}^{x} W^{\frac{1}{2}} d\hat{x}},$$

where A and B are constants of integration and x_a is some chosen value of x.

THE END