UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: MEK4100 — Mathematical Methods in Mechanics

Day of examination: Thursday 5. December 2018

Examination hours: 09.00 – 13.00

This problem set consists of 3 pages.

Appendices: Formula sheet

Permitted aids: Mathematical Handbook, by K. Rottmann.
Approved calculator

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

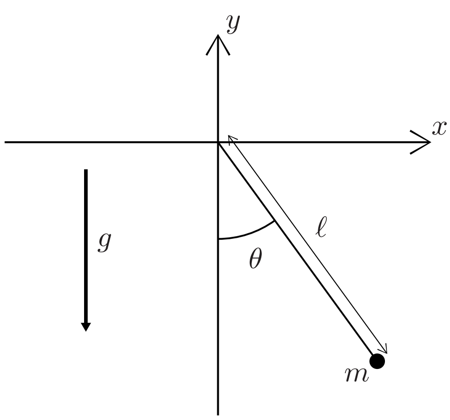
Problem 1 (weight 20%)

The initial value problem

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + y = \epsilon y \left(1 - \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2\right), \quad y(0) = 1, \quad \frac{\mathrm{d}y(0)}{\mathrm{d}t} = 0,$$

has a periodic solution. We assume that ϵ is constant and small. Use a perturbation method to find the solutions for y and the frequency of the oscillations through order ϵ (the two first terms).

Problem 2 (weight 20%)



A mathematical pendulum, in the gravity field, is defined in the figure. Choose the angle of excursion, θ, as generalized coordinate.

(Continued on page 2.)

2a (weight 10%)

Find the Lagrange function and the Lagrange equation in this case.

2b (weight 5%)

Does the Lagrange equations inherit any first integrals ? If that is the case, give a physical interpretation.

2c (weight 5%)

Find the Hamiltonian.

Problem 3 (weight 30%)

A second order differential equation is given as

$$\epsilon(y'' + q(x)y') + W(x)y = 0, \tag{1}$$

where $\epsilon \ll 1$, both q and W are positive everywhere, and x is the free variable. In this problem we will develop the WKB technique for (1).

3a (weight 20%)

A transformation applied to (1) yields the following Ricatti equation for k

$$\epsilon(k' + k^2 + qk) + W = 0. \tag{2}$$

Do this transformation. Then apply the technique of dominant balance to find the first two terms in a perturbation series for k.

3b (weight 10%)

Use the results from the preceding sub-problem to find the full approximate solution for y.

Problem 4 (weight 30%)

In a chemical reaction hydrogen bromide is produced from hydrogen and bromine according to

$$H_2 + Br_2 \rightarrow 2HBr$$

At $t^* = 0$ we start with a concentration C_x of bromine (Br_2), a concentration C_y of hydrogen (H_2) and no hydrogen bromide. Denoting the concentrations of bromine, hydrogen and hydrogen bromide by x^*, y^* and z^*, respectively, the reaction is governed by the equations

$$2x^* + z^* = 2C_x, \quad 2y^* + z^* = 2C_y, \quad \frac{\mathrm{d}z^*}{\mathrm{d}t^*} = k\frac{y^*(x^*)^{\frac{3}{2}}}{x^* + mz^*},$$

where k and m are constants. The first two equations describe the conservation of numbers of bromine and hydrogen atoms respectively, while the last is due to a slightly complicated reaction mechanism. All the concentrations have the same unit, we need not be concerned with the appropriate definition. It is clear that $0 \leq x^* \leq C_x$, $0 \leq y^* \leq C_y$ and $0 \leq z^* \leq 2C_y$. In the following we assume that $C_y/C_x \ll 1$.

(Continued on page 3.)

4a (weight 10%)

Rescale the problem and eliminate variables to obtain an equation set

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{(1-\frac{1}{2}z)(1-\frac{1}{2}\epsilon z)^{\frac{3}{2}}}{1+(m-\frac{1}{2})\epsilon z}, \quad z(0)=0. \tag{3}$$

where $\epsilon \ll 1$.

4b (weight 20%)

The differential equation in (3) is separable, but solving it as a separable equation involves a cumbersome integral and we will abstain from using this method. Instead you are asked to find the first two terms in a perturbation solution for z.

THE END