Mandatory Assignment 1, MEK4300, Fall 2018, due by Wednesday 12 September, 16:00 hours.

Note: to pass you must have 8 / 11 subproblems correct. Problem 2c. must be correctly answered in order to pass.

Problem 1.

We consider variants of steady laminar viscous Poiseuille flow between two parallel planes. The planes make an angle θ with the horizontal. The flow is driven by gravity. No-slip conditions apply at the planes where the upper plane is located at $y = h$ and the lower at $y = -h$, see figure 1a below. The motion may be assumed as two-dimensional, with the fluid velocity tangential to the planes, depending on the y-coordinate that is orthogonal to the planes, i.e. $u = u(y)$. The fluid has kinematic viscosity coefficient ν, dynamic viscosity coefficient μ and density ρ. The acceleration of gravity is g.

a. Put down the kinematic boundary condition at $y = \pm h$.

b. Put down the momentum equation in the x-direction and simplify this for the steady flow.

c. Find the velocity profile. Obtain the volume flux.

d. Evaluate the velocity shear and shear stress at the planes.

Problem 2. The gap between the plates are now occupied with two thin fluid layers. The lower has thickness h_1, kinematic viscosity coefficient ν_1, dynamic viscosity coefficient μ_1 and density ρ_1. The similar quantities of the upper fluid are ν_2, μ_2 and ρ_2. See the sketch in figure 1b below.

a. Put down the kinematic boundary condition at the plates, at $y = h_1$, $y = h_2$.

b. Put down the kinematic boundary condition at the interface between the layers, at $y = 0$.

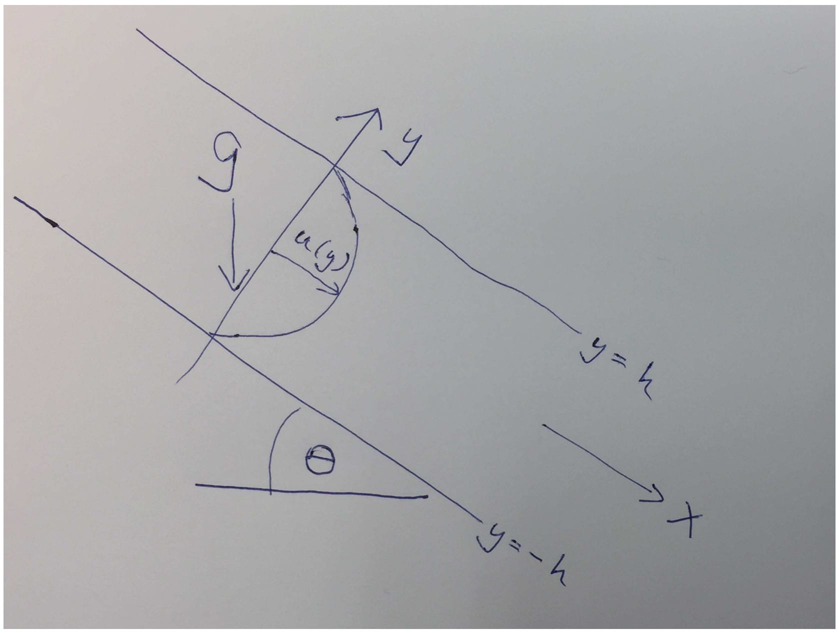
c. Put down the dynamic boundary condition at the interface at $y = 0$.

d. Put down the momentum equation in the x-direction and simplify this for the steady flow, in each of the layers.

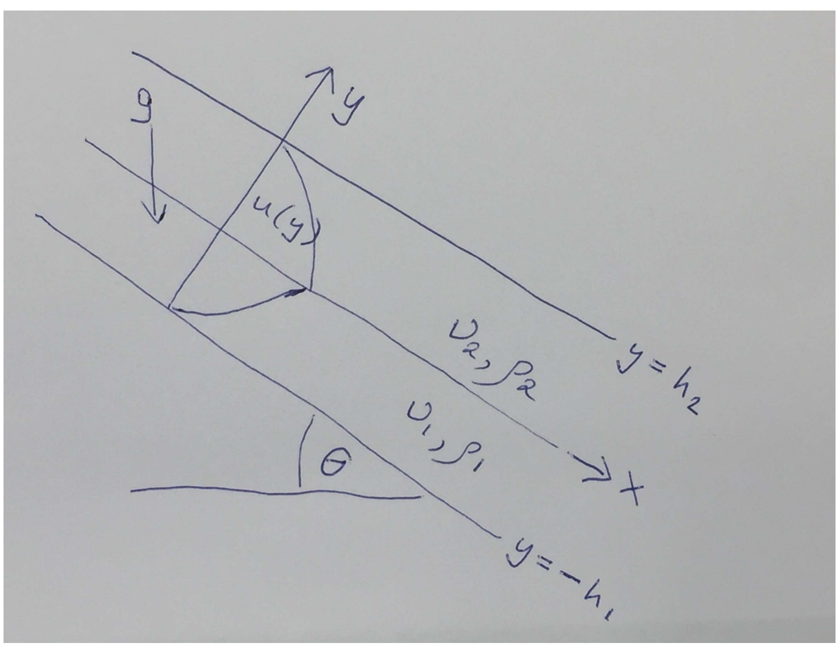
e. Find the velocity profile. Obtain the volume flux in each of the layers.

f. Evaluate the velocity shear and shear stress at the planes and at the interface.

g. Put $\rho_2/\rho_1 = 0.01$ and $\nu_2/\nu_1 = 10$. Put further $h_1 = h_2 = 1$ cm and $\nu_1 = 10^{-6}$ m^{2}s^{-1}. Make a matlab or python script that visualises the velocity profile. Include the visualisation in the report.



a)



b)

Figure 1: Sketch of flow configurations