UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in MEK4300 — Viscous Flow and Turbulence.

Day of examination: Friday, June 12, 2009.

Examination hours: 14.30 – 17.30.

This problem set consists of 4 pages.

Appendices: None.

Permitted aids: Rottmann: Matematische Formelsamlung, approved calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

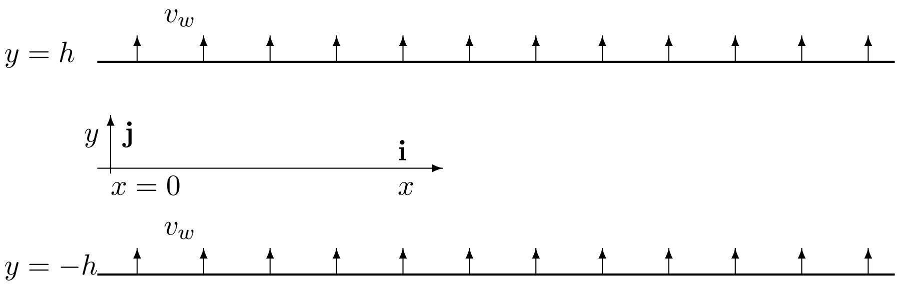

Figure 1 shows schematically the space between two planes $y = h$ and $y = -h$ referred to in the text. The leakage velocity v_w is indicated. The cartesian coordinate system (x, y) used in the problem is also indicated.

We define the space between two planes $y = -h$ and $y = h$ as a duct where flow experiments are carried out. In the first experiment there is a laminar flow in the duct with velocity

$$\mathbf{u} = \mathbf{i}u + \mathbf{j}v \tag{1}$$

The fluid is homogeneous with constant density ρ and constant kinematic viscosity ν. The planes are porous allowing the following boundary conditions

$$v(x, y = -h) = v(x, y = h) = v_w > 0 \tag{2}$$

where v_w is a constant, while

$$u(x, y = -h) = u(x, y = h) = 0 \tag{3}$$

The flow is time independent and fully developed with

$$\nabla p = -\mathbf{i}\rho\beta, \quad \beta > 0 \tag{4}$$

where β is constant.

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a) Find the velocity components u og v. (The algebraic equations determining the constants of integration in the solution for u, are *not* required solved.)

The same duct is used for another flow experiment where the boundary conditions for v are changed to

$$v(x, y = -h) = -v_w \tag{5}$$

$$v(x, y = h) = v_w \tag{6}$$

while the boundary conditions for u still are $u(x, y = \pm h) = 0$.

b) Find find the volume flow rate $Q(x) = \int_{-h}^{h} u(x, y)dy$ given that $Q(x = 0) = Q_0$.

The stream function associated with (u, v) is $\psi(x, y) = g(x)f(y)$.

c) Define the velocity components u og v expressed in terms of the stream function and find $g(x)$ presupposed $f(h) = 1$ and $f(-h) = -1$.

d) Explain what the difference $\psi(x, h) - \psi(x, -h)$ represents physically.

Problem 2

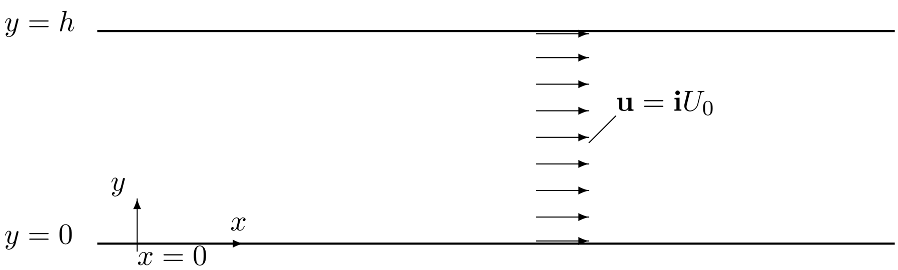

Figure 2 shows schematically the space between two planes $y = 0$ and $y = h$ referred in the text. The planes have temperature distributions as given in the text. The cartesian coordinate system (x, y) referred in the text is shown in the figure.

The space between two planes $y = 0$ and $y = h$ is filled with a *non-viscous* fluid with constant density ρ and constant thermal conductivity k. The fluid is flowing through the space with constant velocity $\mathbf{u} = \mathbf{i}U_0$. The planes have the following temperature distributions

$$T(x, y = 0) = T_0 \tag{7}$$

$$T(x < 0, y = h) = T_0 + \Delta T \tag{8}$$

$$T(x \geq 0, y = h) = T_0 \tag{9}$$

where ΔT is a constant. The temperature distribution in the fluid for $x \leq 0$ is regarded known and given as

$$T(x \leq 0, 0 < y < h) = T_0 + \Delta T \, \frac{y}{h} \tag{10}$$

The development of a temperature field is generally governed by the following equation

$$\rho c_p \frac{DT}{Dt} = k\nabla^2 T + \Phi \tag{11}$$

(Continued on page 3.)

where Φ is the dissipation function. We suppose here that $|\rho c_p U_0 \frac{\partial T}{\partial x}| >> k|\frac{\partial^2 T}{\partial x^2}|$.

a) Explain why the temperature field in the fluid for $x > 0$ in the problem considerd here, can be described approximately by the following equation

$$\rho c_p U_0 \frac{\partial T}{\partial x} = k \frac{\partial^2 T}{\partial y^2} \tag{12}$$

b) Find the temperature distribution in the fluid for $x > 0$.

Hint: It might be useful to know that the function $H(\eta)$ given as

$$H(\eta) = \eta \; for \; -1 < \eta < 1 \tag{13}$$

$$H(\eta) = 0 \; for \; \eta = \pm 1 \tag{14}$$

$$H(\eta \pm 2) = H(\eta), \; \forall \; \eta \tag{15}$$

can be expressed by

$$H(\eta) = \frac{2}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{m} \sin(m\pi\eta) \tag{16}$$

Problem 3

The velocity and pressure fields in a strictly stationary turbulent flow field are denoted $u_i(x_j, t)$ og $p(x_j, t)$ where $i = 1, 2, 3$ and $j = 1, 2, 3$. The fluid is homogeneous with constant density ρ and constant dynamic viscosity μ. The fields may be decomposed into mean fields $U_i(x_j)$, $P(x_j)$ and fluctuating fields $u'_i(x_j, t)$, $p'(x_j, t)$ so that

$$u_i(x_j, t) = U_i(x_j) + u'_i(x_j, t) \tag{17}$$

$$p(x_j.t) = P(x_j) + p'(x_j, t) \tag{18}$$

a) Derive Reynolds avereaged Navier-Stokes equations and define Reynolds stress tensor.

b) Derive a balance equation for the kinetic energy $\frac{1}{2}U_iU_i$ associated with the mean flow and define the production of turbulent kinetic energy.

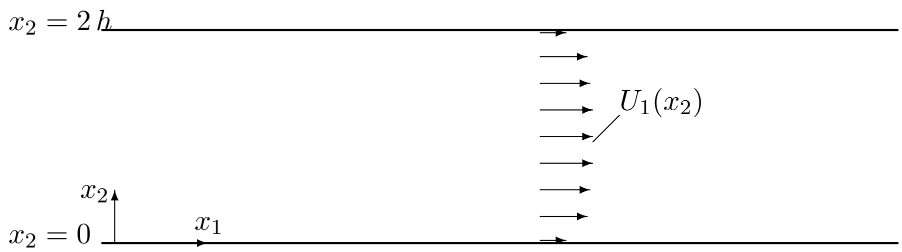

Figure 3 shows schematically the space between two planes $x_2 = 0$ and $x_2 = 2\,h$ referred to in the text. The velocity profile $U_1(x_2)$ is indicated. Cartesian coordinates (x_1, x_2) referred to in the text is also indicated.

(Continued on page 4.)

There is a fully developed turbulent fluid flow in the space between two planes $x_2 = 0$ og $x_2 = 2h$ (as indicated in figure 3) driven by the pressure drop

$$\frac{\partial P}{\partial x_1} = -\alpha \tag{19}$$

where α is a positive constant.

c) Find the shear stress distribution $\tau_{12}(x_2; \alpha; h) \equiv \mu \frac{dU_1}{dx_2} - \rho \overline{u_2' u_1'}$.

   Hint: Observe that τ_{12} can be expressed as a function of x_2 with α and h as parameters.

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