UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: MEK4300/9300 — Viscous Flow and Turbulence.

Day of examination: Friday, June 11, 2010.

Examination hours: 9.00 – 12.00.

This problem set consists of 3 pages.

Appendices: None.

Permitted aids: Rottmann: Matematische Formelsamlung, approved calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

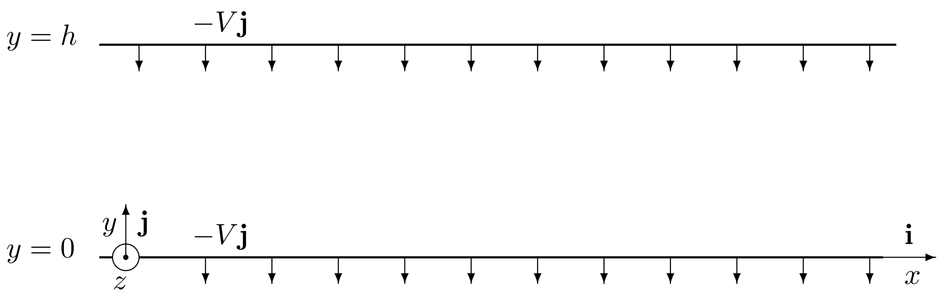

Figure 1 shows schematically the space between two planes $y = h$, $y = 0$ referred to in the text. The leakage velocity $-V\mathbf{j}$ referred to in the text is indicated. The cartesian (x, y, z)-coordinate system used in the problem, is also shown in the figure.

A homogeneous newtonian fluid with constant density ρ and constant kinematic viscosity ν is flowing through the space between two planes $y = 0$ and $y = h$ as indicated in figure 1. The velocity of the fluid, $\mathbf{u} = \mathbf{i}u + \mathbf{j}v$, is induced by the following boundary conditions

$$u(x, y = 0, z) = 0 \tag{1}$$

$$u(x, y = h, z) = U \tag{2}$$

$$v(x, y = 0, z) = v(x, y = h, z) = -V \tag{3}$$

$$w(x, y = 0, z) = w(x, y = h, z) = 0 \tag{4}$$

The flow is fully developed. There is no other motion in the fluid than that induced by the given boundary conditions, and $V > 0$. We consider a control volume bounded by the channel walls $y = 0$ and $y = h$, and the

(Continued on page 2.)

mathematically defined planes $x = 0$ and $x = a$, $z = 0$ and $z = b$. The stress tensor $\mathcal{P}$ is given on the boundary of the control volume as the following

$$\mathcal{P}(0 \leq x \leq a, y = 0, 0 \leq z \leq b) = -p_0\mathcal{E} + (\mathbf{ij} + \mathbf{ji})\tau_0 \tag{5}$$

$$\mathcal{P}(0 \leq x \leq a, y = h, 0 \leq z \leq b) = -p_0\mathcal{E} + (\mathbf{ij} + \mathbf{ji})\tau_1 \tag{6}$$

$$\mathcal{P}(x = 0, 0 \leq y \leq h, 0 \leq z \leq b) = \mathcal{P}(x = a, 0 \leq y \leq h, 0 \leq z \leq b) \tag{7}$$

$$\mathcal{P}(0 \leq x \leq a, 0 \leq y \leq h, z = 0) = \mathcal{P}(0 \leq x \leq a, 0 \leq y \leq h, z = b) \tag{8}$$

where

$$\mathcal{E} = \mathbf{ii} + \mathbf{jj} + \mathbf{kk} \tag{9}$$

and p_0, τ_0 and τ_1 are constants.

a) Find the x-component of the integral momentum balance equation using the given boundary conditions.

b) Find the velocity components u and v.

The temperature on the plane $y = 0$ is T_0 (constant). The temperature on the plane $y = h$ is

$$T(x, y = h, z, t) = T_0 + \Delta T \sin(\omega t) \tag{10}$$

where ΔT is a constant. The temperature equation is

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa \nabla^2 T + \phi(y) \tag{11}$$

c) Neglect the dissipation $\phi(y)$ and verify when using

$$T(y, t) = T_0 + \Delta T \Theta(y, t) \tag{12}$$

that the solution of $\Theta(y, t)$ can be written

$$\Theta(y, t) = \Re\{[A_1 \exp(\alpha_1 y) + A_2 \exp(\alpha_2 y)] \exp(i\omega t)\} \tag{13}$$

$$\alpha_1 = \frac{-V + \sqrt{V^2 + 4i\omega\kappa}}{2\kappa} \tag{14}$$

$$\alpha_2 = \frac{-V - \sqrt{V^2 + 4i\omega\kappa}}{2\kappa} \tag{15}$$

Problem 2

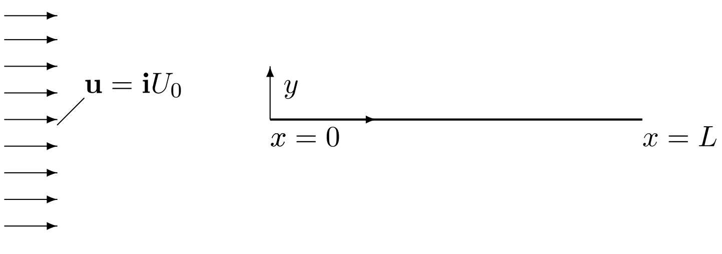

Figure 2 indicate schematically the flat plate at $y = 0$ and $0 < x < L$, referred to in the text.

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A fully developed viscous boundary layer flow is established along a flat plate, as indicated in figure 2. The x-component of the velocity field in the boundary layer, $u(x,y)$, is approximated by

$$u(x,y) = U_0\,(1 - \exp(-\alpha(x)y)) \tag{16}$$

U_0 is constant, and $\alpha(x) = \frac{1}{5}\sqrt{\frac{U_0}{\nu x}}$, where ν is the constant kinematic viscosity of the fluid.

a) Find y-component $v(x,y)$ of the velocity field in the boundary layer.

b) Find the displacement thickness $\delta^*(x)$ in the boundary layer.

Problem 3

In a newtonian fluid with constant density ρ and constant kinematic viscosity ν, a statistically stationary turbulent state of motion has been established. The velocity field is denoted $\mathbf{u}(\mathbf{x},t)$, and the pressure field is denoted $p(\mathbf{x},t)$. Reynolds decomposition of the fields should be introduced so that

$$\mathbf{U}(\mathbf{x}) = \lim_{T\to\infty}\left[\frac{1}{T}\int_0^T \mathbf{u}(\mathbf{x},t)dt\right] \tag{17}$$

$$P(\mathbf{x}) = \lim_{T\to\infty}\left[\frac{1}{T}\int_0^T p(\mathbf{x},t)dt\right] \tag{18}$$

and thereby $\mathbf{u}(\mathbf{x},t) = \mathbf{U}(\mathbf{x}) + \mathbf{u}'(\mathbf{x},t)$, og $p(\mathbf{x},t) = P(\mathbf{x}) + p'(\mathbf{x},t)$.

a) Find the Reynolds averaged momentum equation and the Reynolds averaged continuity equation.

b) Find the equation for the mean flow kinetic energy pr mass unit $\frac{1}{2}U_iU_i$.

c) Verify that the equation for the kinetic energy of the turbulence $\frac{1}{2}\overline{u_i'u_i'}$ can be written

$$\begin{aligned} U_j\frac{\partial}{\partial x_j}\left[\frac{1}{2}\overline{u_i'u_i'}\right] &= -\frac{\partial}{\partial x_j}\left[\overline{p'u_j'} + \frac{1}{2}\overline{u_i'u_i'u_j'} - 2\nu\overline{u_i's_{ij}'}\right] \\ &\quad - \overline{u_i'u_j'}S_{ij} - 2\nu\overline{s_{ij}'s_{ij}'} \end{aligned} \tag{19}$$

where $S_{ij} = \frac{1}{2}(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i})$ and $s_{ij}' = \frac{1}{2}(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i})$.

d) What does $\overline{u_i'u_j'}S_{ij}$ and $2\nu\overline{s_{ij}'s_{ij}'}$ represent physically.

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