# UNIVERSITY OF OSLO

## Faculty of Mathematics and Natural Sciences

Examination in: MEK4300/9300 — Viscous flow og turbulence

Day of examination: Friday 15. June 2012

Examination hours: 9.00 – 13.00

This problem set consists of 3 pages.

Appendices: Formula sheet

Permitted aids: Rottmann: Matematische Formelsamlung, certified calculator

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

## Problem 1 Turbulence (weight 15%)

Derive the Reynolds averaged Navier-Stokes (RANS) equations for an incompressible, Newtonian fluid and identify the Reynolds stress tensor. You may use, without proof, that $\mathbf{v} \cdot \nabla \mathbf{v} = \nabla \cdot (\mathbf{v}\mathbf{v})$ for any divergence-free velocity field $\mathbf{v}$.

## Problem 2 Gravity driven viscous flow (weight 35%)

A film of liquid is flowing down at the outside of a vertical cylinder of radius $a$, under the action of gravity. We assume that the thickness of the fluid film, $b - a$, is constant along (and around) the cylinder and that the flow is stationary and radially symmetric. At the surface of the fluid there is an external pressure, $p_0$, while the shear stress is negligible. The cylinder is at rest.

### 2a (weight 10%)

Formulate equations and boundary conditions, invoking the simplifying assumptions.

### 2b (weight 20%)

Find the pressure and the velocity distribution.

### 2c (weight 5%)

Calculate the drag ($D$) on the cylinder per height. Show that $D$ obeys

$$D = \rho g A,$$

*(Continued on page 2.)*

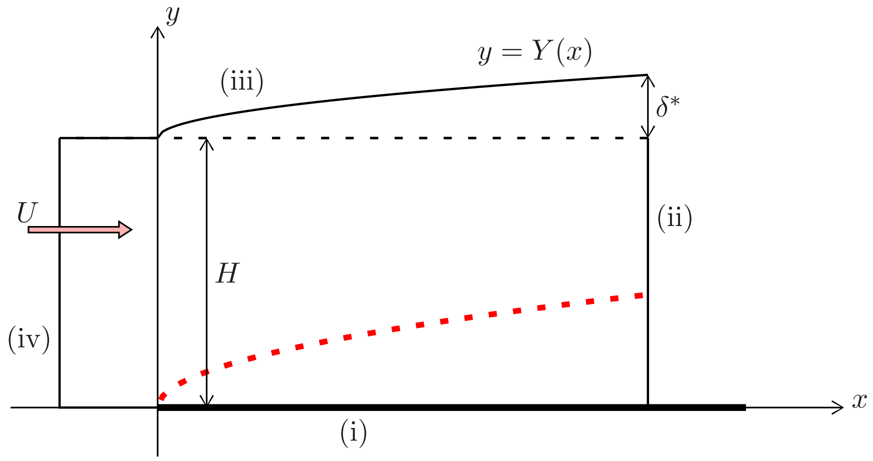



Figure 1: A control volume for mass balance analysis. The upper boundary of the volume, $y = Y(x)$, is a streamline outside the boundary layer. Also the line $y = H$ is outside the boundary layer, which is indicated by the bold dashes.

where $A$ is the cross-sectional area of the fluid. Explain this relation physically. Correspondingly, the total dissipation per height and the volume flux, $Q$, through the cross-sectional area, fulfill the relation

$$\iint_A \Phi \, dx \, dy = \rho g Q,$$

where $\Phi$ is the dissipation pr volume. Do not (!) calculate $\Phi$ and $Q$, but explain the relation physically.

## Problem 3 Boundary layer (weight 50%)

In this problem we consider the Blasius boundary layer, which develops along a semi-infinite plate, corresponding to the positive $x$-axis, with the leading edge at the origin. The fluid is homogeneous and incompressible and the background flow is $U\mathbf{i}$, where $\mathbf{i}$ is the unit vector in the $x$-direction. Moreover, the $y$-axis is aligned normal to the plate and the velocity components in the $x$ and $y$ directions are $u$ and $v$, respectively.

### 3a (weight 10%)

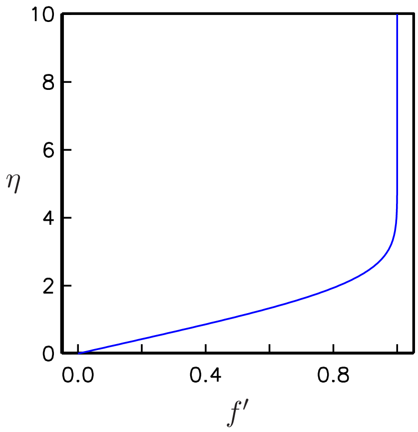
A control volume is depicted in figure 1. The sides are numbered (i), (ii), (iii) and (iv). The boundary (iii) corresponds to a streamline outside the boundary layer. Use the mass balance argument to show that the displacement thickness is given by

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

### 3b (weight 20%)

Give the boundary layer equations and boundary conditions for the Blasius flow. A derivation is not required, but the main differences from the full Navier-Stokes equations should be listed.

*(Continued on page 3.)*




| $\eta$ | $f$ | $f'$ | $f''$ |
|---|---|---|---|
| 0.000000 | 0.000000 | 0.000000 | 0.469600 |
| 0.500000 | 0.058643 | 0.234227 | 0.465030 |
| 1.000000 | 0.232990 | 0.460633 | 0.434379 |
| 1.500000 | 0.515032 | 0.661474 | 0.361805 |
| 2.000000 | 0.886797 | 0.816695 | 0.255669 |
| 2.500000 | 1.322440 | 0.916808 | 0.147475 |
| 3.725000 | 2.509840 | 0.994972 | 0.014234 |
| 4.975000 | 3.758240 | 0.999929 | 0.000283 |
| 7.475000 | 6.258220 | 1.000000 | 0.000000 |

Figure 2: The form function of the Blasius profile.

**3c** (weight 10%)

We assume the existence of a similarity solution

$$u = Uf'(\eta), \quad \eta = \frac{y}{\Delta(x)},$$

where the functions $f$ and $\Delta$ are to be determined. Find a corresponding expression for $v$, show that we may employ $\Delta = \sqrt{2\nu x/U}$ and find an ordinary differential equation, with boundary equations, for $f$. The solution for $f$ is depicted and tabulated in figure 2. However, a discussion of its computation is not required.

**3d** (weight 10%)

Use the preceding results to find explicit expressions for the displacement thickness, the shear stress at the plate and the drag (per width) of a section of length $D$ from the front of the plate. Discuss briefly the validity of these expressions.

The End