UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: MEK4300/9300 — Viscous flow og turbulence

Day of examination: Friday 14. June 2013

Examination hours: 9.00 – 13.00

This problem set consists of 4 pages.

Appendices: None

Permitted aids: Rottmann: Matematische Formelsamlung, certified calculator

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1 Turbulence (weight 35%)

Fluid flow is often described mathematically through the Navier-Stokes equations:

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{f}, \tag{1}$$

$$\nabla \cdot \boldsymbol{u} = 0, \tag{2}$$

where $\boldsymbol{u}$, p, ρ, ν and $\boldsymbol{f}$ are the velocity vector, pressure, density, kinematic viscosity and volume forces respectively.

1a (weight 5%)

Which two physical laws were used in the derivation of the Navier-Stokes equations? What other assumptions have been made?

1b (weight 10%)

Introduce Reynolds decomposition of velocity and pressure and derive from (1) og (2) the Reynolds Averaged Navier-Stokes (RANS) equations.

1c (weight 10%)

Turbulent flows are often 'defined' through a list of characteristic properties. Give at least 5 characteristic properties that define turbulence.

1d (weight 10%)

Explain in words how it is possible to derive a transport equation for $\overline{u_i u_j}$ (Note, not $\overline{u_i' u_j'}$). Explain also how $\overline{u_i' u_j'}$ can be modelled through an algebraic expression.

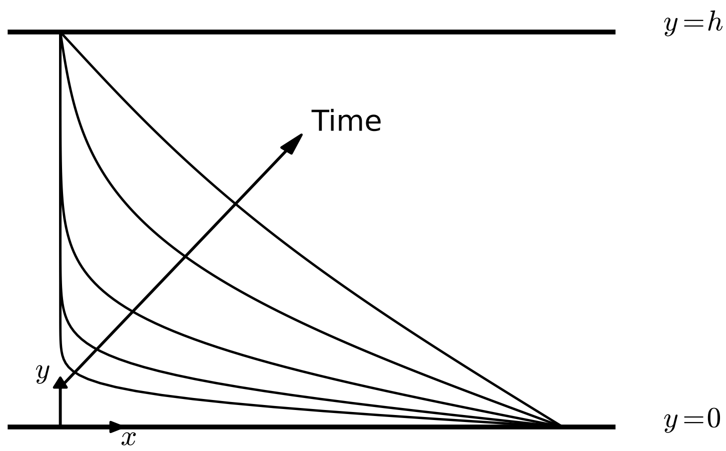

Figure 1: Sketch of the flow between two plates located at $y = 0$ og $y = h$. The thin lines illustrate how the flowfield evolves in time. The velocity is then plotted along the x-axis as a function of y.

Problem 2 Transient flow between two parallel plates (weight 25%)

Consider a uniform, straight flow of an incompressible Newtonian fluid between two parallel plates of infinite extension in x and z-directions. The lower plate is located at $y = 0$ and the upper at $y = h$. The velocity vector is $\boldsymbol{u} = (u(y,t), 0, 0)$. The fluid between the plates is initially at rest. At $t = 0$ the velocity of the lower plate is suddenly accelerated to $u(0,t) = U$, while the upper is kept still. The flow field is illustrated in Figur 1.

2a (weight 10%)

Formulate the equations, including boundary and initial conditions for this problem.

2b (weight 15%)

Find the velocity $u(y,t)$. (Hint: Introduce $v(y,t) = u(y,t) - U(1 - y/h)$ and solve the homogeneous problem for $v(y,t)$ first. Also, $\int_0^h \sin^2(n\pi y/h)dy = h/2$ and $\int_0^h (1 - y/h)\sin(n\pi y/h)dy = h/(n\pi)$, for $n = 1, 2, 3\ldots$).

Problem 3 Laminar boundary layer (weight 40%)

We consider a laminar boundary layer that evolves over a plate lying at position $y = 0$ in the plane spanned by the x- and z-axes. The boundary layer evolves from the origin and grows in the positive x-direction. The boundary layer is generated by the outer flow $U\mathbf{i}$, where $\mathbf{i}$ is the unit normal vector in the x-direction. The velocitycomponents in x and y directions are, respectively, u and v. Two common quantities often used in defining boundary layers are

$$\delta^*(x) = \int_0^{y\to\infty} \left(1 - \frac{u}{U}\right) \mathrm{d}y,$$
$$\theta(x) = \int_0^{y\to\infty} \frac{u}{U}\left(1 - \frac{u}{U}\right) \mathrm{d}y.$$

3a (weight 10%)

What are the quantities δ^* and θ called and from which two physical laws are they derived?

3b (weight 10%)

Assume that the velocity u is known and given as

$$u(y;\delta) = U \sin\left(\frac{\pi y}{2\delta}\right), \tag{3}$$

where $\delta(x)$ is the actual boundary layer thickness (defined through $u(x,\delta) = 0.99U$). The skinfriction coefficient, C_f, is defined as

$$C_f = \frac{\tau_w(x)}{0.5\rho U^2} = 2\frac{\mathrm{d}\theta}{\mathrm{d}x}, \tag{4}$$

where $\tau_w(x) = \mu \mathrm{d}u/\mathrm{d}y$ at $y = 0$. Use this to find δ^* and θ expressed in terms of x. Feel free to simplify the results using $\mathrm{Re}_x = \rho U x/\mu$.

3c (weight 10%)

The laminar boundary layer equations can be derived from the Navier-Stokes equations by proper scaling and subsequent elimination of small terms in the limit of large Reynolds numbers ($\mathrm{Re} = \rho U L/\mu$). Use the following normalizations

$$\overline{x} = \frac{x}{L} \qquad \overline{u} = \frac{u}{U} \qquad \overline{v} = \frac{v}{U}\sqrt{Re}$$
$$\overline{y} = \frac{y}{L}\sqrt{Re} \qquad \overline{t} = \frac{tU}{L} \qquad \overline{p} = \frac{p}{\rho U^2}$$

and show that the boundary layer equations can be written as

$$\frac{\partial \overline{u}}{\partial \overline{t}} + \overline{u}\frac{\partial \overline{u}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{u}}{\partial \overline{y}} = \frac{\partial^2 \overline{u}}{\partial \overline{y}^2} - \frac{\partial \overline{p}}{\partial \overline{x}},$$
$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0,$$
$$\frac{\partial \overline{p}}{\partial \overline{y}} = 0.$$

3d (weight 10%)

We introduce, like Blasius did in 1908, a similarityvariabel $\eta(x, y)$ and a similaritysolution $\psi(x; \eta)$ defined as

$$\eta(x, y) = y\sqrt{\frac{U}{2\nu x}},$$
$$\psi(x, \eta) = \sqrt{2\nu U x} f(\eta),$$

where $f(\eta)$ is an unknown function to be determined and ψ is the streamfunction defined as

$$u = \frac{\partial \psi}{\partial y} \quad \text{og} \quad v = -\frac{\partial \psi}{\partial x}.$$

Derive, starting from the boundary layer equations, the Blasius equation

$$f^{'''} + f f^{''} = 0.$$

Formulate also the boundary conditions for these equations.

The End