UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: MEK4300/9300 — Viscous flow and turbulence

Day of examination: Wednesday 10. June 2015

Examination hours: 14.30 – 18.30

This problem set consists of 3 pages.

Appendices: None

Permitted aids: Rottmann: Matematische Formelsamlung

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1 Laminar and turbulent boundary layers (weight 50%)

We consider a boundary layer forming over a flat plate located at $y = 0$ and starting at $x = 0$, as shown in Figure 1. The flow is assumed to be incompressible and the fluid is Newtonian at constant temperature. The outer flow is given as $U\boldsymbol{i}$, where U is constant and $\boldsymbol{i}$ is the unit normal in the x direction. We assume first that the flow is laminar.

1a (weight 10 %)

Use conservation of mass and show, with the help of von Kármán's control volume (see Fig. 1), that

$$\delta^* = \int_0^{y\to\infty} \left(1 - \frac{u(y)}{U}\right) dy \tag{1}$$

The quantity δ^* is shown in Figure 1. What is δ^* called?

1b (weight 10%)

Use conservation of momentum in the x direction and show, with the help of the control volume, that

$$\theta = \frac{\text{Drag}}{\rho U^2} = \int_0^{y\to\infty} \frac{u(y)}{U}\left(1 - \frac{u(y)}{U}\right) dy \tag{2}$$

What is θ^1 called?

1c (weight 15 %)

Assume now that the flow is turbulent. Reynolds decomposition of the velocity vector, $\boldsymbol{u} = (u, v, w)$, is given as $\boldsymbol{u}(\boldsymbol{x}, t) = \overline{\boldsymbol{u}}(\boldsymbol{x}) + \boldsymbol{u}'(\boldsymbol{x}, t)$, where

(Continued on page 2.)

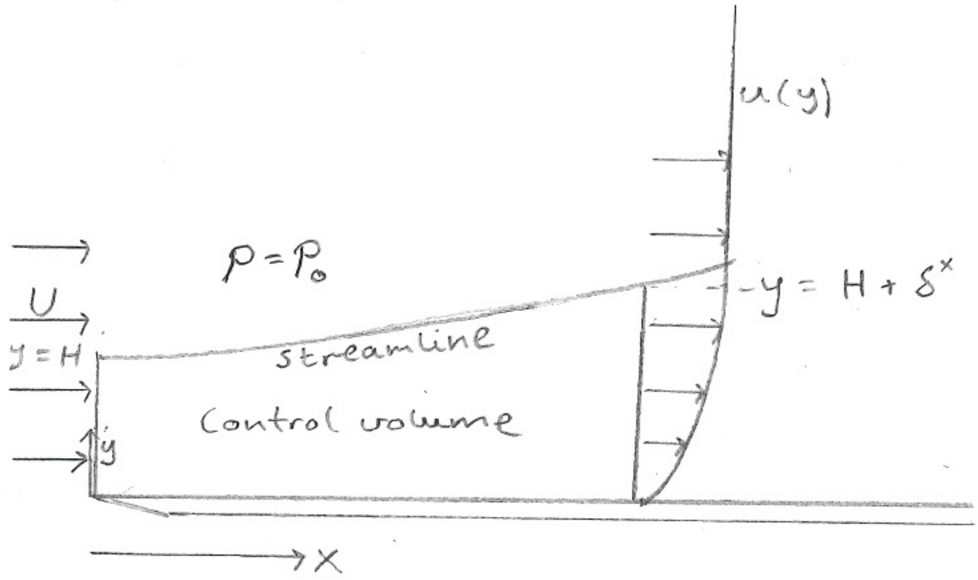

Figure 1: Development of a boundary layer over a flat plate. von Kármán's control volume is shown. The velocity U and the pressure given by P_0 are both constant.

$\overline{\boldsymbol{u}}$ and $\boldsymbol{u}'$ are, respectively, the mean velocity vector and the fluctuation about this mean. Derive the Reynolds averaged Navier Stokes equations governing this problem. Simplify as much as possible. Define also the boundary conditions for a given x (disregarding the inlet and outlet). Hint: $\overline{v} << \overline{u}$, $\frac{\partial}{\partial x} << \frac{\partial}{\partial y}$ and $\frac{\partial}{\partial z} = 0$ (same as for laminar boundary layers).

1d (weight 15%)

The turbulent (Reynolds) shear stress is given as $-\rho\overline{u'v'}$, where ρ is density. Explain how $\rho\overline{u'v'}$ can be modelled and explain what is meant by a *complete turbulence model.* Also explain (using drawings if you will) why we should expect that $\overline{u'v'} \leq 0$ throughout the boundary layer.

Problem 2 Mixed Poiseuille-Couette flow between two parallel plates (weight 50%)

We consider a mixed Poiseuille-Couette flow of an incompressible Newtonian fluid between two parallel plates of infinite extension in the plane spanned by the x and z axes. The plates are located at $y = \pm 1$ and initially the flow between the plates is steady and the plates have constant velocities $u(1, t) = U$ and $u(-1, t) = -U$, where $t < 0$ and U is a positive constant. The velocity vector is given as $\boldsymbol{u} = (u(y, t), 0, 0)$. At time $t = 0$ a pressure gradient is suddenly applied $\frac{\partial p}{\partial x} = -\rho\beta$, where β is a constant.

2a (weight 5%)

What is meant, respectively, by Couette and Poiseuille flows?

(Continued on page 3.)

2b (weight 10%)

Formulate the fully simplified set of equations describing this problem, with initial and boundary conditions.

2c (weight 5%)

Find the steady state solution $u_s(y)$ that the flow will reach eventually.

2d (weight 15%)

Find the velocity $u(y, t)$. Hint: Use the steady solution from 2c and solve for $v(y, t) = u(y, t) - u_s(y)$. You may also use that $\int_{-1}^{1} \cos(\lambda_k y)(1 - y^2)\, dy = 4(-1)^{k-1}/\lambda_k^3$, where $\lambda_k = (2k - 1)\pi/2$, for $k = 1, 2, 3 \ldots$.

2e (weight 15%)

Implement a numerical solution to the problem using Python/FEniCS. Also, implement the exact analytical solution found in 2d and show how the numerical error may be estimated at every time step.

The End