12th November, 2023

MEK4350 / MEK9350

Mandatory assignment 2 of 2

Submission deadline

Thursday 30th November 2023, 14:30 at Canvas canvas.uio.no.

Instructions

Note that you have **one attempt** to pass the assignment. This means that there are no second attempts.

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) well before the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

NB! The full problem set is quite large. However, you only have to analyse one of the datasets in problem 1. That is all! You are recommended to do our irregular laboratory wave, at least one of the ocean wave records, and the turbulent sound in problem 1. If you can make at least an attempt to do problem 2 and/or 3, that would be great, but this is not required!

Problem 1. Time series of laboratory waves, ocean waves and turbulent sound

There are four different types of time series that you can analyse:

I The irregular wave time series we got from our laboratory experiment on 20th September 2023: `proberun4`.

II Ocean swell time series from a buoy on deep water in the Bay of Biscay (download `1D.zip` and extract file `BayOfBiscay.mat`)

III Ocean storm waves, either the Ekofisk time series from the Knud storm (download `rec_ekofiskL_hz2_20180921_134000.hz2`) or the Draupner New Year Wave time series (either download `New_Year_Wave.txt`, or download `1D.zip` and extract file `NewYearWave.mat`). For the following analysis the Ekofisk time series will give a more representative result.

IV Turbulent sound time series from our laboratory (download `case1.wav`).

For each record do the following:

a. Plot a reasonable part of the time series.

b. Compute the significant wave height H_s from the standard deviation.

c. Compute the frequency resolution Δf and the Nyquist frequency.

d. Estimate the variance/power spectrum $S(f)$ from the time series. Make sure the normalization criterion is satisfied!

   (In real life you would probably do this with `pwelch` or some other "black box", but for this exercise please carry out the Fourier transforms by explicit calls to `fft` / `ifft`.)

e. Compute the significant wave height H_s from the power spectrum $S(f)$.

f. Plot the one-sided spectrum for $f \geq 0$ for frequencies up to the Nyquist frequency. Use dimensional axes, the first axis should be given in Hertz (Hz). The plot should have doubly logarithmic axes, i.e. it should be made with `loglog`.

g. Does the tail of the spectrum obey a power law? Determine which power law, i.e. determine the slope of the tail in the loglog plot. Do this by guessing a suitable power law, plot it into the loglog plot and check that it fits (approximately).

Problem 2. A snapshot of the surface elevation of a wave field

Download `2D.zip` and extract the following files:

{`Bimodal`, `Swell`, `WindSea` } / `ReWFLin00000_0000?.mat`

where `?` is a number between 0 and 9. These records are synthetic data supposed to correspond to snapshots of the ocean surface.

Each file is written in MATLAB format and contains the following:

- `x`: sampling points along the x-axis in meters.
- `y`: sampling points along the y-axis in meters.
- `eta`: surface elevation $\eta(x, y)$ in meters.

Do the following:

a. Compute the resolution in Fourier space $(\Delta k_x, \Delta k_y)$ and the respective Nyquist wave numbers. Use units from the SI-system.

b. Estimate the two-dimensional spectrum $F^{(2)}(\boldsymbol{k})$.

   Hint: `fft2` / `ifft2` carry out 2D Discrete Fourier Transforms.

c. Plot the spectrum $F^{(2)}(k_x, k_y)$. Make sure the wave vector $\mathbf{0}$ is in the middle of the plot.

d. Is it possible to identify in which direction these waves propagate?

e. Compute the significant wave height H_s for each record with the following techniques:

   - From the standard deviation of the surface elevation.
   - From the two-dimensional spectrum $F^{(2)}(\boldsymbol{k})$.

f. Can you identify that the records respectively correspond to a wind sea, a swell and a bimodal sea state?

Problem 3. Spatio–temporal surface elevation wave fields

Download `3D.zip` and extract the following three files:

`Record3D_1.mat`, `Record3D_2.mat`, `Record3D_3.mat`

Each file is written in MATLAB format and contains the following:

- `dt`: sampling time Δt in seconds.
- `dx`: sampling interval along the x-axis Δx in meters.
- `dy`: sampling interval along the y-axis Δy in meters.
- `nt`: number of elements in time.
- `nx`: number of elements along the x-axis.
- `ny`: number of elements along the y-axis.
- `waves3d`: surface elevation $\eta(x, y, t)$ in meters.

Do the following:

a. Compute the resolution in Fourier space $(\Delta k_x, \Delta k_y, \Delta\omega)$ and the respective Nyquist wave numbers and frequency. Use units from the SI-system.

b. Estimate the three-dimensional spectrum $F^{(2+1)}(\boldsymbol{k}, \omega)$.

   Hint: `fftn` / `ifftn` carry out n-dimensional Discrete Fourier Transforms.

c. Plot the spectrum $F^{(2+1)}(k_x, k_y, \omega)$ for selected cross-sections of constant frequency, ω = constant, with $\omega \geq 0.5$ rad/s.

d. Plot the spectrum in the cross-section $F^{(2+1)}(0, k_y, \omega)$.

e. Plot the spectrum in the cross-section $F^{(2+1)}(k_x, 0, \omega)$.

f. Is it possible to identify the dispersion relation from the plots above?

g. Compute and plot the unambiguous wave number spectrum $F_+^{(2)}(\boldsymbol{k})$.

h. Compute the significant wave height H_s for each record with the following techniques:

   - From the standard deviation of the surface elevation.
   - From the tree-dimensional spectrum $F^{(2+1)}(\boldsymbol{k}, \omega)$.
   - From the two-dimensional unambiguous wave vector spectrum $F_+^{(2)}(\boldsymbol{k})$.

i. Looking at these results, which record corresponds to wind sea, swell and a bimodal sea state?