MEK 4480 Assignment 2 (Hand in assignment) – Problems Stokes flow

1. Suppose $\mathbf{u}$ and $\mathbf{u}'$ are two solutions of the Stokes equations that satisfy the same boundary conditions on S, which is the boundary to the volume V. Show that $2\mu \int_V (e'_{ij} - e_{ij})(e'_{ij} - e_{ij}) dV = 2\mu \int_V (\mathbf{e}' - \mathbf{e})(\mathbf{e}' - \mathbf{e}) dV = 0$. What does this mean? ($e_{ij}$ is the rate of strain tensor.)

2. Consider a closed region of fluid with volume V bounded by the surface S. Suppose $\mathbf{u}$ and $\mathbf{u}'$ are two solutions of the Stokes equations, with the respective stress field $\mathbf{T}$ and $\mathbf{T}'$. Show that $\oint_S \mathbf{v} \cdot (\mathbf{T}' \cdot \mathbf{n}) dS = \oint_S \mathbf{v}' \cdot (\mathbf{T} \cdot \mathbf{n}) dS$ where $\mathbf{n}$ is the normal to the surface. What does this proof illustrate?

3. The vector potential is defined as $\mathbf{A} = \frac{\psi}{r \sin(\theta)} \mathbf{e}_\phi$, show how the differential operator $E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin(\theta)}{r^2} \frac{\partial}{\partial \theta} (\frac{1}{\sin\theta} \frac{\partial \psi}{\partial \theta}) = \frac{\partial^2}{\partial r^2} + \frac{1-\eta^2}{r^2} \frac{\partial^2}{\partial \eta^2}$ with $\eta = cos(\theta)$ appears when deriving the equation for the vorticity $\omega = -\frac{1}{r\sqrt{1-\eta^2}} E^2 \psi \mathbf{e}_\phi$ and the stream function $E^2(E^2\psi) = 0$ in spherical coordinates.

4. Prove equation 7-132 (Hints: i) How do you express the unit vectors in spherical coordinates in terms of the unit vectors in rectangular coordinates? ii) Consider the limit when R^* is very large iii) Find how to describe the Stokes equations in spherical coordinates (check the text book)? How to compute the pressure and its contribution to the force? (Maple/ Mathematica my help the final integration once the problem is defined.)

5. Derive Stokes law for drag force on a sphere with radius a in a flow with constant velocity along the symmetry axis. How does the force change if we have free slip on the substrate of the sphere i.e. the limit of a bubble, $e_{r\theta} = 0$ at $r = a$?

6. We have the sphere with mass m of radius a that sediments by gravity $-g$ in z-direction in a quiescent fluid with viscosity μ and density ρ. i) Determine the constant sedimentation velocity in z-direction ii) Assume the sphere is exposed to a side wind with constant speed U_x along the x-direction. Find the velocity of the sphere along the two directions, x and z.

7. Consider a liquid droplet with viscosity $\hat{\mu}$ that rises at a constant velocity $U\mathbf{e_z}$ through a viscous fluid with viscosity μ, where the flow in both phases are described by Stokes flow. Assume no deformation of the droplet. i) What are the boundary conditions at the free surface ii) Find the stream function inside $\hat{\psi}$ and outside ψ the droplet and plot the streamlines. iii) Find the drag force on the droplet. Discuss the limits when $\lambda = \hat{\mu}/\mu \to 0$ and $\lambda = \hat{\mu}/\mu \to \infty$