MEK 4480 – Assignment 3

1. Consider an interface of two immicible fluids described by $r = R(\theta, \phi, t)$ in spherical coordinates, how do you express $(\frac{\partial R}{\partial t})_{\theta,\phi}$ in terms of the velocity components u_r, u_θ and u_ϕ?

2. A straw of radius R is immersed into a liquid through an air-liquid interface. The straw is coated with a material that causes the liquid to contact the surface of the straw with a contact angle θ. Show that the liquid will rise up the straw a distance

$$H = \frac{2\gamma \cos\theta}{\rho g R}, \tag{1}$$

   where γ is the interfacial surface tension between the liquid and the air and ρ is the fluid density. (see Problem 2-30 in the Leal's book, p.107-108)

3. In the lecture, we derived the thin film equation from conservation of mass. The thin film equation reads

$$\frac{\partial h}{\partial t} + \frac{\partial U_d h}{\partial x} = 0, \tag{2}$$

   and

$$U_d = \frac{1}{h}\int_0^h U_x dy. \tag{3}$$

   The evolution of the interface can also be derived from the kinematic condition, we then obtain

$$\frac{\partial h}{\partial t} = [U_y - U_x \frac{\partial h}{\partial x}]|_{y=h}. \tag{4}$$

   Show that they are equivalent.