MEK 4480 – Assignment 4

1. When we derived the thin film equation, gravity was neglected. Suppose a liquid film is flowing on a tilted solid surface, see Fig. 1. The angle between the solid surface and the horizon is α. Derive the thin film equation when gravity is also playing a role. Suppose a large droplet is spreading on a horizontal surface (neglect the surface tension), find the similarity solution of the spreading droplet for a fixed volume.

2. Similarity solutions for adhesive elastohydrodynamic touchdown: we consider an elastic sheet (described by $h(x,t)$) on top of a solid surface with a fluid in between (imagine replacing the fluid/air interface in capillarity by the elastic sheet). Force balance on the elastic sheet gives

$$p(x,t) = B\frac{\partial^4 h(x,t)}{\partial x^4} + \frac{A}{3h^3(x,t)}, \tag{1}$$

where $p(x,t)$ is the fluid pressure, B is the bending stiffness and $\frac{A}{3h^3(x,t)}$ is the van der Waals adhesion pressure. We consider the elastic interface to be a thin sheet (or beam) with small deflections, with a normal shear force ($N(x,t)$) and torque ($M(x,t)$). For small deflections $\partial h/\partial x \ll 1$ the elastic torque is approximated by the curvature $M(x,t) = B\frac{\partial^2 h}{\partial x^2}$. Torque balance implies that $N = \frac{\partial M}{\partial x}$, while normal stress balance gives us $\frac{\partial^2 M}{\partial x^2} = p - \frac{A}{3h^3(x,t)}$ as the density of the liquid and the sheet is assumed to be identical and there is no inertia or tension in the sheet. Note that for fluid/air interface, the surface tension is $\gamma\frac{\partial^2 h(x,t)}{\partial x^2}$ and tangential to the surface. What is the thin film equation in this case? Suppose the elastic sheet is perturbed slightly from a flat shape, determine the stability condition. What is the dispersion relation (i.e. relation between the wave number and frequency)? If the initial profile of the sheet is unstable, the van der Waals attraction will then pull it further close to the solid surface. This touchdown process can be described by a similarity solution. Defining the similarity variable as

$$H(\eta) = \frac{h(x,t)}{(t_c - t)^{\alpha}}, \quad \eta = \frac{x - x_c}{(t_c - t)^{\beta}}, \tag{2}$$

derive the ordinary differential equation (ODE) for $H(\eta)$ from the thin film equation. Here t_c and x_c are the time and the position when the elastic sheet touches the solid surface respectively. What are the boundary conditions for the ODE?

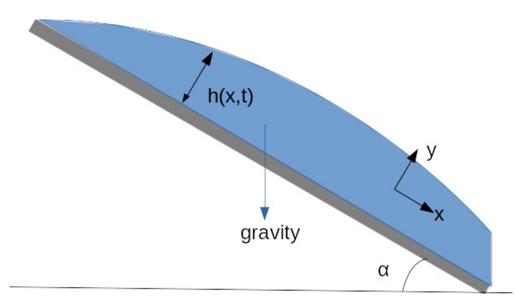

FIG. 1: A fluid film flowing on a tilted surface.