MEK4480 — project 2

Obligatorisk oppgave: project2

Innleveringsfrist

Friday 14th may 2021, klokken 14:30 i Canvas (canvas.uio.no).

Instruksjoner

Du velger selv om du skriver besvarelsen for hånd og scanner besvarelsen eller om du skriver løsningen direkte inn på datamaskin (for eksempel ved bruk av LaTeX). Besvarelsen skal leveres som én PDF-fil. Scannede ark må være godt lesbare. Besvarelsen skal inneholde navn, emne og oblignummer.

Det forventes at man har en klar og ryddig besvarelse med tydelige begrunnelser. Husk å inkludere alle relevante plott og figurer. Studenter som ikke får sin opprinnelige besvarelse godkjent, men som har gjort et reelt forsøk på å løse oppgavene, vil få én mulighet til å levere en revidert besvarelse. Samarbeid og alle slags hjelpemidler er tillatt, men den innleverte besvarelsen skal være skrevet av deg og reflektere din forståelse av stoffet. Er vi i tvil om du virkelig har forstått det du har levert inn, kan vi be deg om en muntlig redegjørelse.

I oppgaver der du blir bedt om å programmere må du legge ved programkoden og levere den sammen med resten av besvarelsen. Det er viktig at programkoden du leverer inneholder et kjøreeksempel, slik at det er lett å se hvilket resultat programmet gir.

Søknad om utsettelse av innleveringsfrist

Hvis du blir syk eller av andre grunner trenger å søke om utsettelse av innleveringsfristen, må du ta kontakt med studieadministrasjonen ved Matematisk institutt (7. etasje i Niels Henrik Abels hus, e-post: studieinfo@math.uio.no) i god tid før innleveringsfristen.

For å få adgang til avsluttende eksamen i dette emnet, må man bestå alle obligatoriske oppgaver i ett og samme semester.

For fullstendige retningslinjer for innlevering av obligatoriske oppgaver, se her:

www.uio.no/studier/admin/obligatoriske-aktiviteter/mn-math-oblig.html

Regler for obligatoriske oppgaver i MEK4480:

Du skal levere EN PDF fil i Devilry — ikke flere filer, ikke noe annet enn PDF. Du må presentere en av delene i Obligen i vår regnetime uken etter innleveringsfristen, som blir avtalt i forelesning.

LYKKE TIL!

Time (s)	3.2	14.5	18.2	33.3	73.4	92.1	156.4
R (cm)	0.098	0.109	0.111	0.118	0.131	0.137	0.142

Tabell 1: Experimental data of a silicone oil droplet spreading on a smooth wafer. The surface tension of the silicone oil/air interface is 0.04 N/m. The viscosity of the silicone oil is 0.01 Pa·s . The size of the droplet is 0.033 mm^3. The contact angle is $\theta_m = 0$ rad.

1 Spreading of a viscous droplet over a smooth solid surface

When a viscous liquid droplet is placed to be in contact with a smooth solid surface, the droplet will spread in order to minimize the surface energy. For completely wetting surfaces (i.e. the contact angle $\theta_m = 0$) and small droplets (i.e. gravity is neglected), the spreading follows Tanner's law. In this regime, the droplet spreading radius $R(t)$ scales with time t as $R(t) = kt^{1/10}$, here k is a prefactor that depends on the viscosity of the fluid η, the interfacial surface tension γ and the volume of the droplet V. In this project, we will study the spreading of a droplet on a solid surface.

1.1 The physical properties of the liquid and the surfaces

The spreading depends on the properties of the liquid and the surface (i.e. viscosity, droplet radius, surface tension, contact angle and gravity). Based on these material properties find the non-dimensional numbers for the spreading process. What is the characteristic spreading velocity (how do you construct a velocity scale from the parameters)? For what droplet size can gravity be neglected?

1.2 Post-processing the experimental data

In this project, you are given a set of data from an experimental measurement, see table 1. You need to describe the dynamics. One quantity to characterize spreading is the droplet radius $R(t)$, how does it depend on time? Plot the extracted radius ($R(t)$) as a function of time in logarithmic axis. Discuss the results.

1.3 Theory and numerical simulation

1. If a no-slip boundary condition is imposed at the solid/liquid boundary, hydrodynamic theories predicts that a droplet would never spread over a solid surface as the viscous dissipation tends to infinity at the moving contact line. A contact line is the common boundary where three phases meet, i.e. liquid, air and solid. To demonstrate the problem of infinite viscous dissipation at a moving contact line, we consider a 2D liquid wedge moving with a constant velocity U over a solid surface. The angle between the liquid/air interface and the solid surface is denoted as θ. Suppose θ is small, the flow inside the liquid is a parabolic thin film flow. Assuming the liquid/air interface is described by $h = \theta x$ (this is a weak assumption, we will see in the next question that the interface is indeed curved near the contact line), show that the viscous dissipation in the liquid around the moving contact line (i.e. $x = 0$) is infinite .

2. From the above calculation we see that the viscous stress is strong near the contact line. The interface $h(x)$ is thus highly deformed in the contact line region. For the droplet spreading phenomenon, we can separate the droplet into two regions: the contact line region (inner region) and the macroscopic region (outer region). The profile of the interface in the contact line region is a solution of the steady thin film equation (in the moving contact line frame). The steady thin film equation reads

$$h''' = \frac{3Ca}{h^2}, \tag{1}$$

where $Ca \equiv \eta U/\gamma$, U is the contact line velocity. Show that $h = Ax(\ln(x/\lambda))^\beta$ is a solution of the steady thin film equation for $x/\lambda \gg 1$. What are the values of A and β. If we impose a condition that $h' = \theta_m$ (the microscopic contact angle) when $x = \ell_m$, here ℓ_m is a microscopic cutoff length scale for the singularity of the moving contact line, show that $\lambda = \ell_m/(e^{\theta_m^3/9Ca})$. We will then end up with the Cox-Voinov relation which describes the profile in the contact line region, which reads

$$h'^3 = \theta_m^3 + 9Ca\ln(x/\ell_m). \tag{2}$$

3. The viscous stress is weak far away from the contact line, hence the profile of the droplet in the outer region (that is what you observe in the experiment) can be described by a spherical cap with a slowly varying apparent contact angle θ_a (contact angle of the spherical cap). How is the contact radius R of a spherical cap related to the apparent contact angle for a fixed volume V?

4. Matching: We can obtain the Tanner's law by matching the inner and the outer solutions. Note that the Tanner's law is valid for completely wetting (i.e. $\theta_m = 0$). The condition for the matching is $h'(x = L) = \theta_a$, here $L \gg \ell_m$. Derive the Tanner's law based on this matching condition. Hints: the contact line velocity is the time derivative of the contact radius $R(t)$.