Turbulence and hot-wire measurements
Mek 4600

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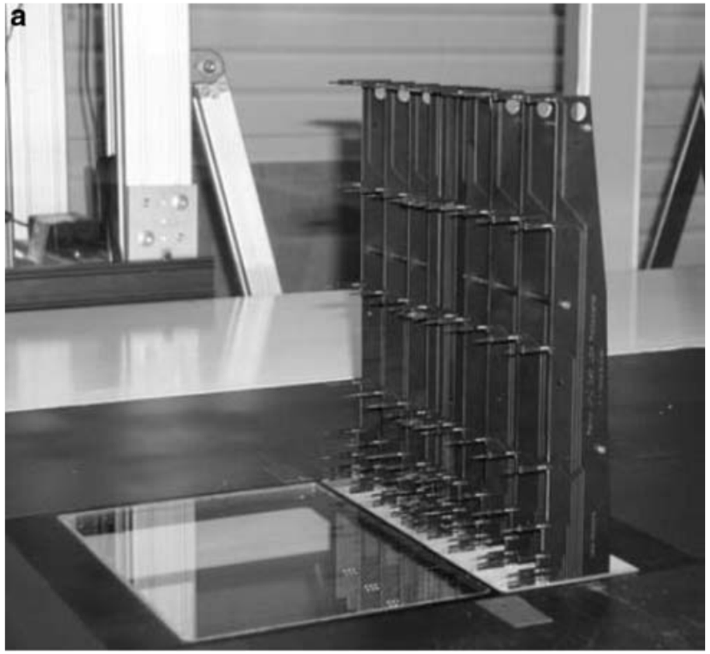
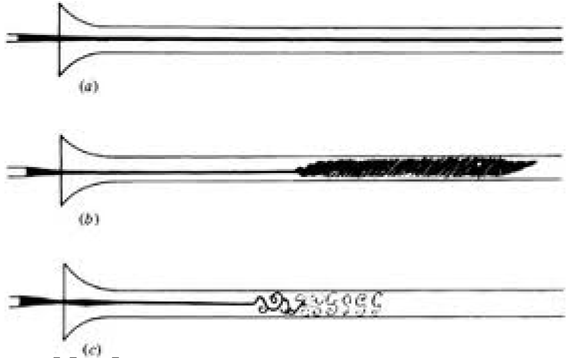


Hot-wire rake of 143 single-wire probes

Turbulence! What is it?



Realizations of solutions to the *governing equations* and *boundary conditions*.

Most engineering flows are turbulent.

Is turbulence still a problem?
What about Navier-Stokes Equations?

C.-L. Navier

Instantaneous Navier-Stokes Equations (NSE):

$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_i} + \upsilon \frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j}$$

G.G. Stokes

Continuity equation:

$$\frac{\partial \tilde{u}_k}{\partial x_k} = 0$$

There are some real disadvantages associated with these equations.

These Navier-Stokes equations are *non-linear* due to convective term: $\tilde{u}_j \, \partial \tilde{u}_i / \partial x_j$

Most solutions of interest are '*random*' (or '*stochastic*') and '*chaotic*' in character.

All scales of motion are important to the dynamics; none are negligible.

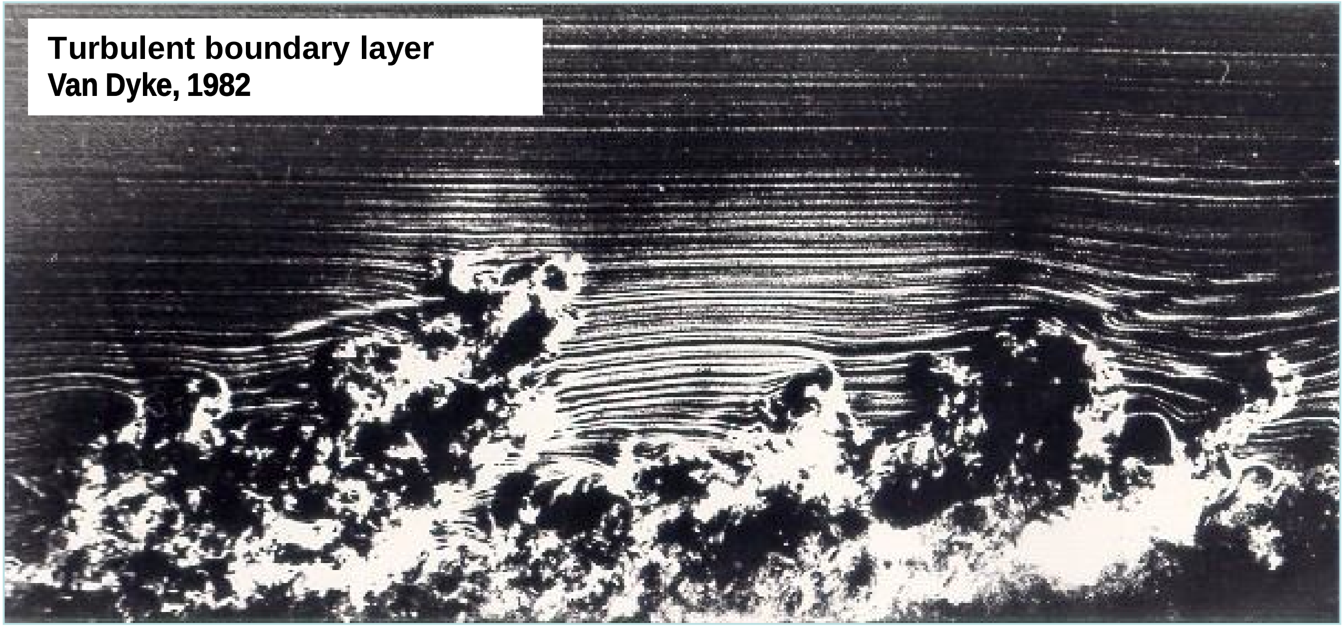
Do we really bother with this?

The answer is YES!

Turbulence is almost everywhere:

- Aero/hydrodynamics
  (Airplanes, ships, submarines, road vehicles, trains)
  (Pipeline, channels, distribution systems)
- Environmental flows
- Industrial processes (chemical and multiphase)
- Combustion
- Energy technology (gas turbines, wind turbines,...)

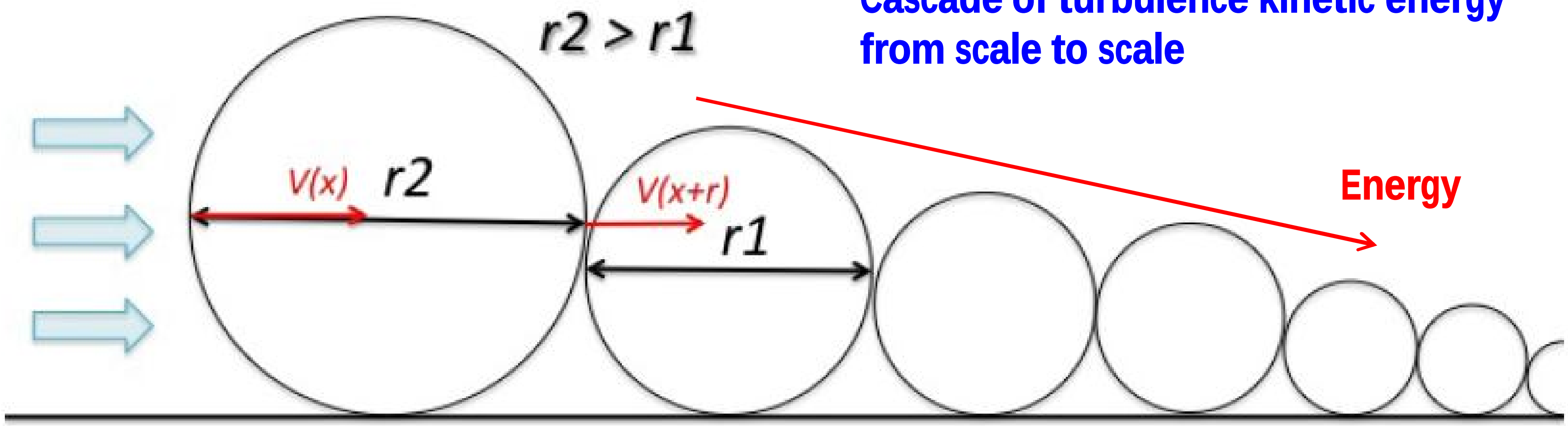
The range of scales in real turbulent flows is enormous... typically 10^5 to 10^{21}



$$\boxed{\mathrm{Re} = \frac{inertia}{viscous}}$$

The higher Re,
the greater the separation of scales.

It is very difficult to measure every scale and will be many decades before computing them directly.



Physical space – Structure functions:

$$v(r) := [u(x+r) - u(x)]$$

Spectral space – Energy spectra:

$$\hat{u}_i(\vec{k},t) = \frac{1}{(2\pi)^3} \int\int\int_{-\infty}^{\infty} u_i(\vec{x},t) e^{-i\vec{k}\cdot\vec{x}} d\vec{x}$$

The primary goal of any turbulence research is to be able to predict or at least model turbulence.

Flow Physics

(DNS & Experiments)

Turbulence Modeling

Flow control

Validation

Application

(**Performance Enhancement** and **Energy Efficiency**)

What can we do then?

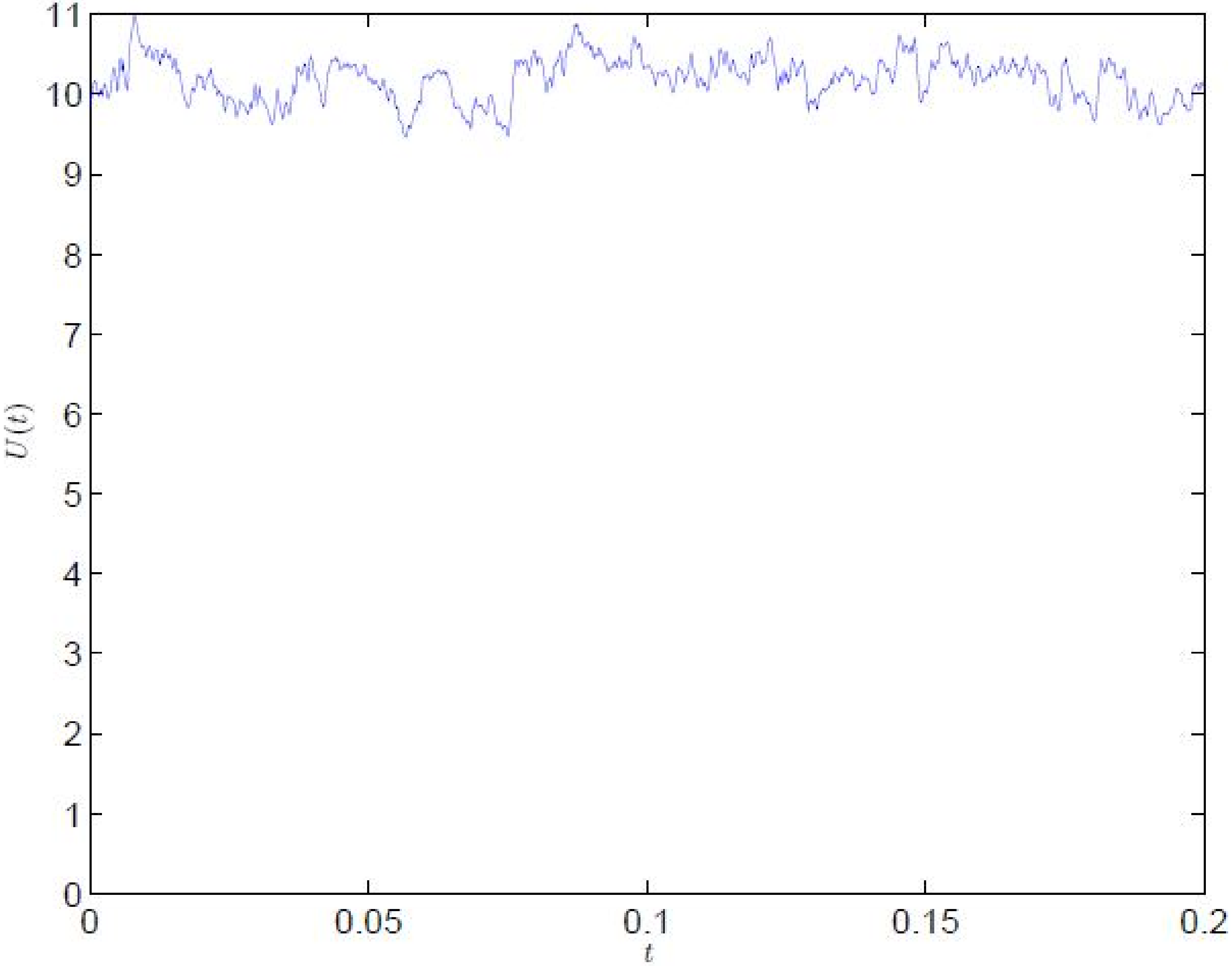
One obvious way is to divide flow into mean and fluctuations (the so called *Reynolds decomposition*).

e.g., mean: $U_i(\vec{x},t) \equiv \left\langle \tilde{u}_i(\vec{x},t) \right\rangle$

fluctuation: $u_i(\vec{x},t) \equiv \tilde{u}_i(\vec{x},t) - U_i(\vec{x},t)$

Ensemble average < > is space and time-dependent.

One has to be careful when computing statistical quantities in turbulence:



Streamwise velocity behind a grid in a wind tunnel

Let's define a time mean estimator as:

$$U_T = \frac{1}{T}\int_0^T U(t)\,dt$$

Let's define the true time mean (average) as

$$\overline{U} \equiv \lim_{T\to\infty} \frac{1}{T}\int_0^T U(t)\,dt$$

Clearly U_T is unbiased since it is straightforward to show that

$$\lim_{T\to\infty} U_T = \overline{U}$$

The question of convergence is as usual addressed by studying the variability.

$$\epsilon_{U_T}^2 = \frac{var\left\{U_T - \overline{U}\right\}}{\overline{U}^2} = \sqrt{\frac{2I}{T}}Tu$$

Example:

To illustrate this we can take an example where the turbulence intensity, Tu, is 20% and the integral time scale is $I = 1.5$ ms. Say that you want an error of at most 1%. Then the total sampling time becomes

$$T = \frac{2I\,(Tu)^2}{\epsilon_{U_T}^2} = \frac{2 \times 1.5 \times 10^{-3} \times 0.2^2}{0.01^2} = 1.2\,\mathrm{s}$$

This corresponds to

$$N_{eff} = \frac{T}{2I} = \frac{1.2}{2 \times 1.5 \times 10^{-3}} = 400$$

Statistically, it does not matter if we take more samples during the 1.2 seconds of sampling. This yields an optimal sampling rate of

$$f_s = \frac{N_{eff}}{T} = \frac{400}{1.2} \approx 333\,\mathrm{Hz}$$

Thus, to ensure that the samples are statistically independent, one has to wait at least two integral time scales between samples.

It is not easy to get the statistics right in turbulence measurements; in particular high order moments.

$$\epsilon_{u^n}^2 = \frac{2I}{T}\frac{var\{u^n\}}{(\overline{u^n})^2} = \frac{2I}{T}\left(\frac{\overline{u^{2n}} - (\overline{u^n})^2}{(\overline{u^n})^2}\right) \tag{4.16}$$

For the second moment we get

$$\epsilon_{u^2}^2 = \frac{2I}{T}\left(\frac{\overline{u^4}}{(\overline{u^2})^2} - 1\right) \tag{4.17}$$

Thus in order to estimate the error of the second moment, we need to know the fourth!. For the third moment (the skewness) we get

$$\epsilon_{u^3}^2 = \frac{2I}{T}\left(\frac{\overline{u^6}}{(\overline{u^3})^2} - 1\right) \tag{4.18}$$

For the fourth moment (the flatness or sometimes called the kurtosis) the error estimate becomes

$$\epsilon_{u^4}^2 = \frac{2I}{T}\left(\frac{\overline{u^8}}{(\overline{u^4})^2} - 1\right) \tag{4.19}$$

Plugging the Reynolds decomposition into the NSE yields the Reynolds Averaged N-S equations (RANS).

Mean momentum equation:

$$\rho\left\{\frac{\partial}{\partial t}+U_j\frac{\partial}{\partial x_j}\right\}U_i=\frac{\partial}{\partial x_i}P+\frac{\partial}{\partial x_j}\left[-\boxed{<\rho u_i u_j>}+\tau_{ij}^{(v)}\right]$$

$<\rho u_i u_j>$ is the so-called **Reynolds stress**.

- It is a *flow property.*
- Working against mean flow gradient and *extracts energy* to turbulence at large scales.

Now we have a closure problem due to $<\rho u_i u_j>$: No new equations, 9 (6 independent) new unknowns

Original "gradient" idea:
(Boussinesq (1877))

$$<u_i u_j> \equiv \nu_t \frac{\partial U_i}{\partial x_j}$$

- 130 years since the turbulent viscosity,
- **k-epsilon** models are just another way to guess.

It has been proven that **simple ideas/approaches do not work** in this problem

Reynolds stress models are another way...

Using Navier-Stokes equations to `build' a set of equations for the Reynolds stress tensor.

$$\frac{\overline{D}}{Dt} < u_i u_k > = \frac{\partial}{\partial x_j}\left[\boxed{-\frac{1}{\rho}(< p u_i \delta_{jk} > + < p u_k \delta_{ij} >)} - < u_i u_k u_j > + \nu \frac{\partial}{\partial x_j} < u_i u_k > \right]$$

$$- < u_i u_j > \frac{\partial U_k}{\partial x_j} - < u_k u_j > \frac{\partial U_i}{\partial x_j}$$

$$- \boxed{\frac{1}{\rho}\left[< p \frac{\partial u_i}{\partial x_k} > + < p \frac{\partial u_k}{\partial x_i} > \right]} - \nu < \frac{\partial u_i}{\partial x_j} \frac{\partial u_k}{x_j} >$$

- The number of unknowns is now **52**, but only 13 eqns!
- Presence of **pressure** presents a huge problem!
  (non-locality)

Anybody who does research in turbulence should keep following points in mind:

The flow at a single point is related to the flow at every other point, and at all previous times **(Triadic Interactions)**.

Even the terms in our averaged equations are **NON-LOCAL** in **both** space and time.

This presents real problems for turbulence models, since all **closures** are **LOCAL**.

How much of these can be measured in the lab?
--- Unfortunately, not much!

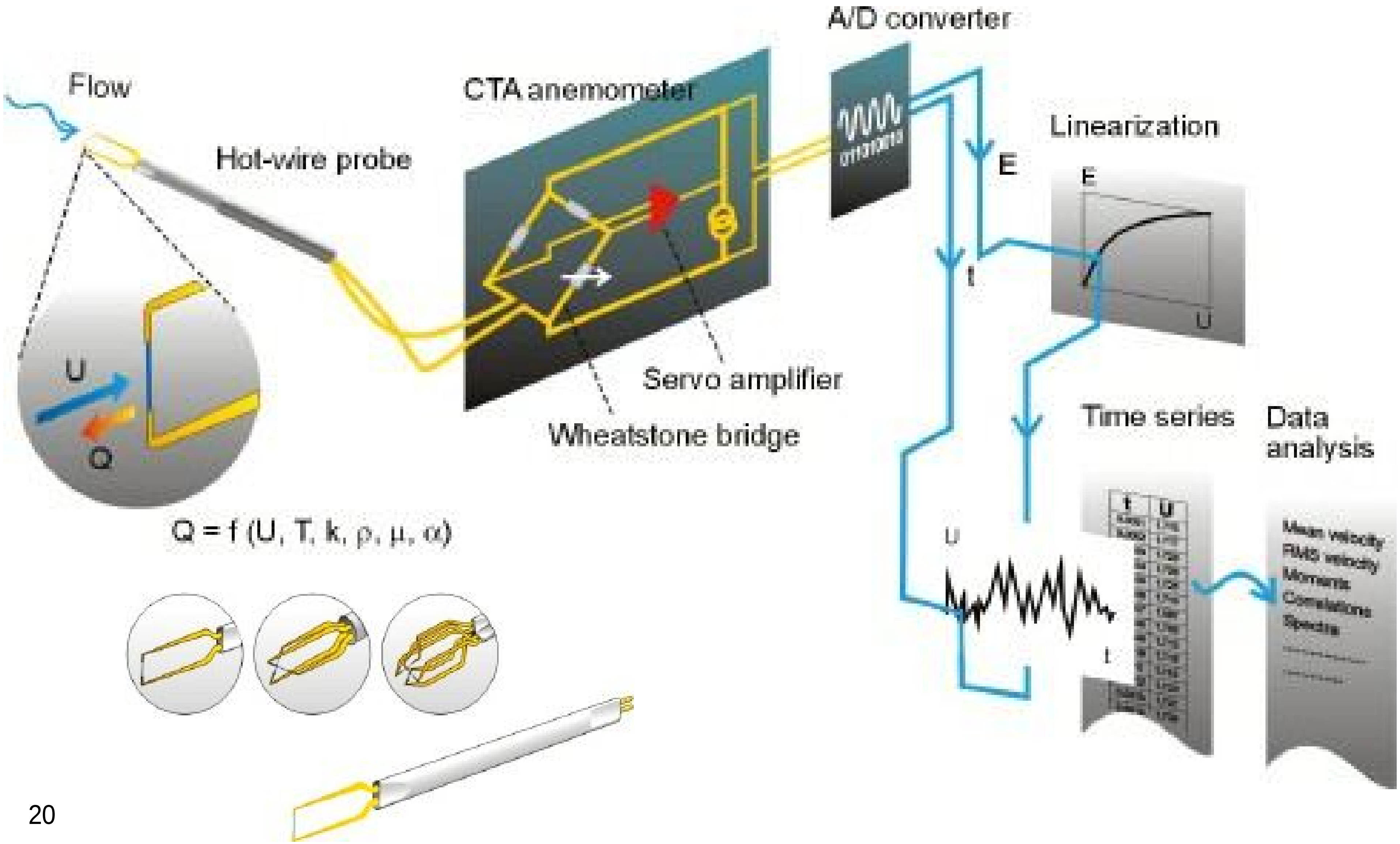
- *Particle Image Velocimetry:*
  - 2, or 3 component of velocity at very high spatial resolution,
  - Most of the systems provide low temporal resolution,
  - Measurement field is often small,
  - Near-wall and low turbulence measurements are very difficult.

- *Laser Doppler Anemometry*
  - Very good at high turbulence measurements,
  - Handles very near-wall region,
  - Single point measurements,
  - Can provide reasonable sampling frequencies.

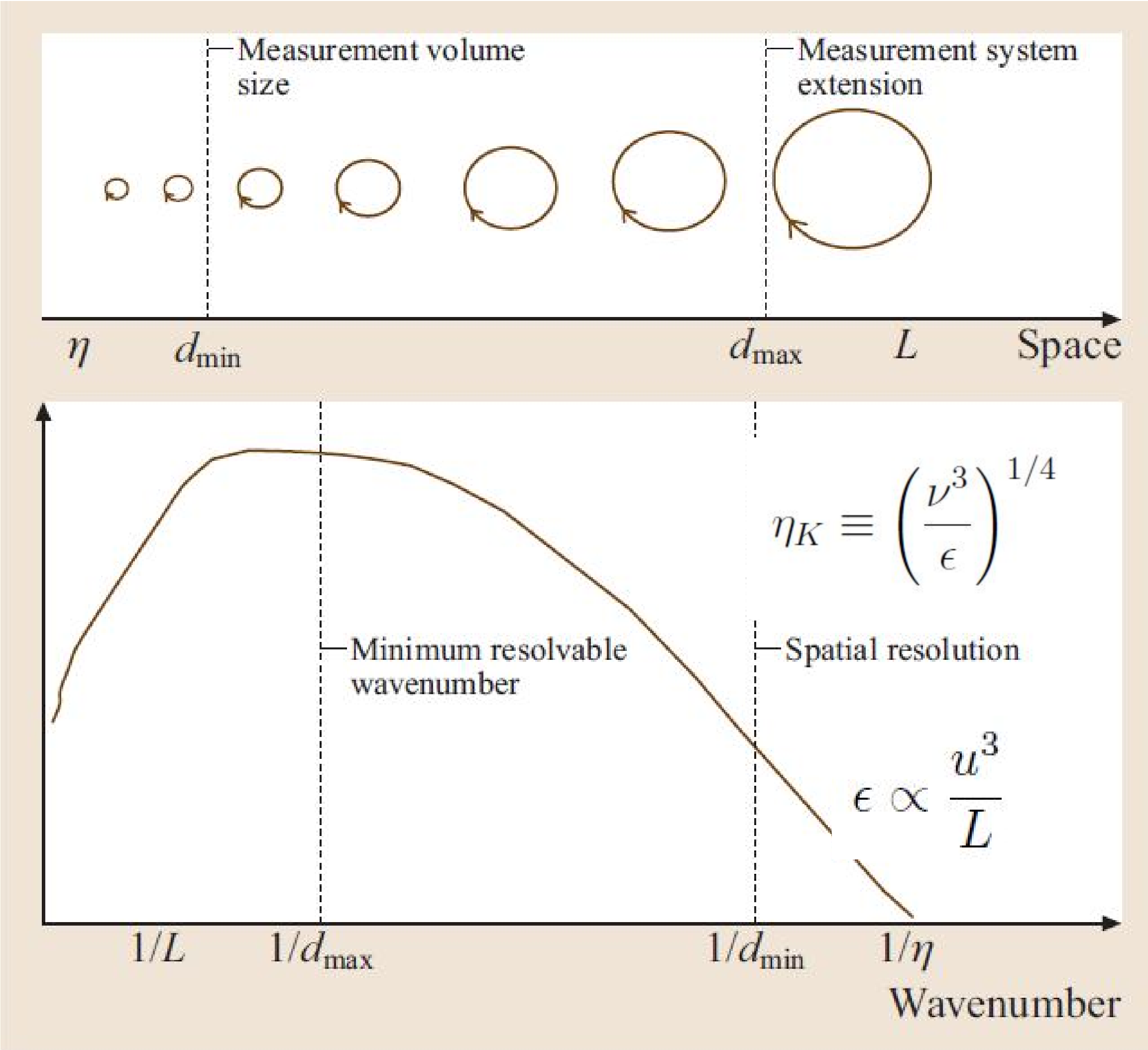
We will focus on the most common measurement methodology used in turbulence reasearch (even today).

- *Hot – wire anemometry:*
  - 1, 2, or 3 component of velocity at very high temporal resolution,
  - Based on heat balance along the sensor element,
  - Single point measurements,
  - Disturbance to the flow,
  - Poor response in high turbulence and recirculation,
  - Cheap compared to the others,
  - Easy to manufacture in-house.

Operating principle (as visualized by Dantec)

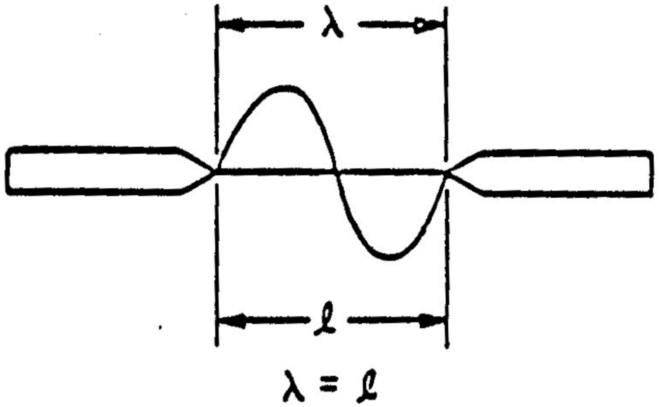


Finite probe size limits the resolvable smallest scale.

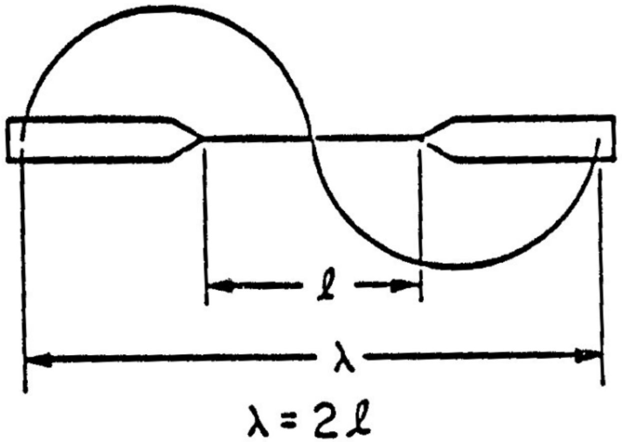


Exp. Fluid Mech. 2007

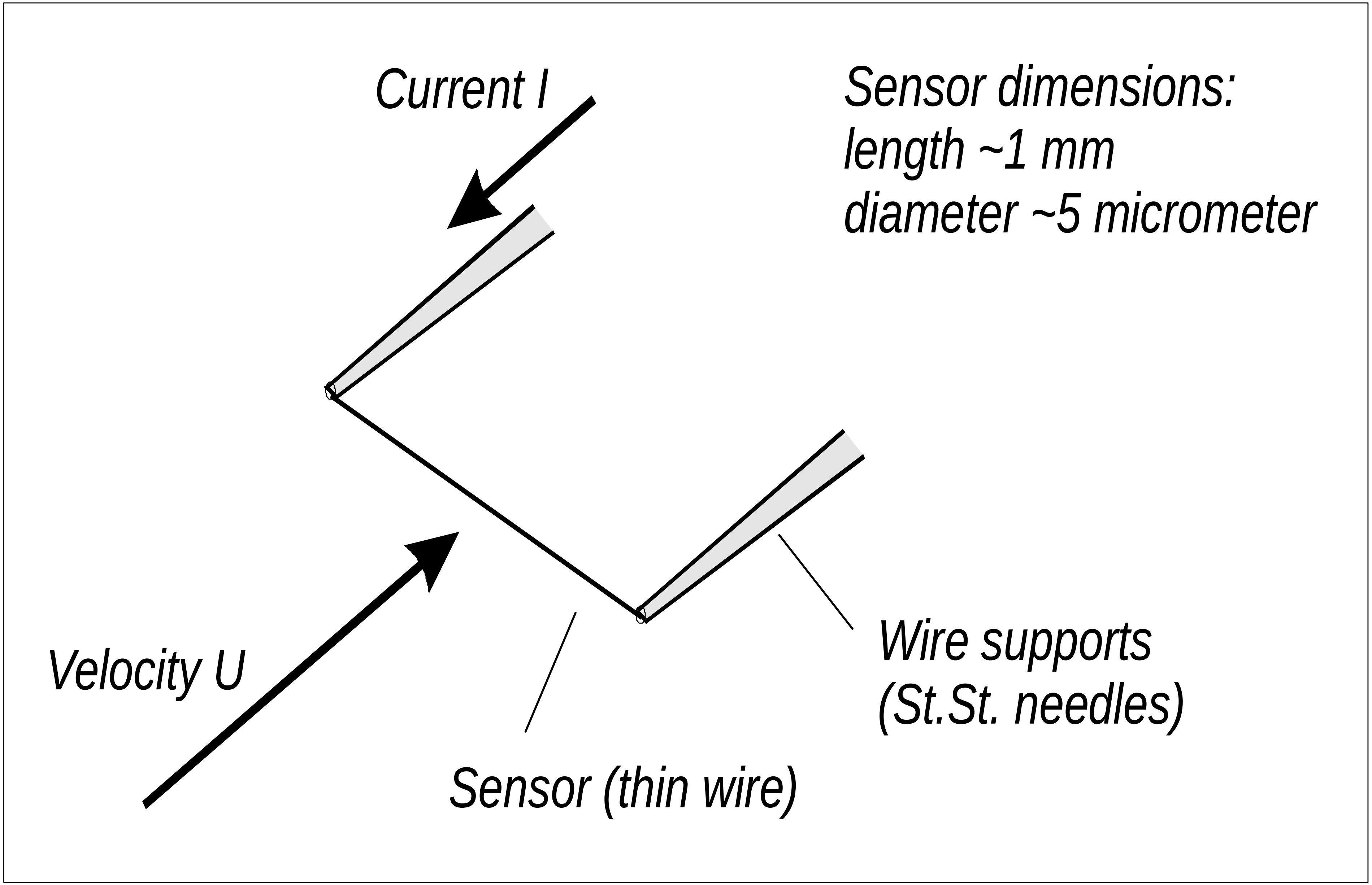
Non-resolvable signal



Resolvable signal



Cut-off frequency in practice : $f_c = \frac{U_c}{2 \times \ell_w}$


Current I
Sensor dimensions:
length ~1 mm
diameter ~5 micrometer
Velocity U
Wire supports
(St.St. needles)
Sensor (thin wire)

All dates back to 1914: L.V. King (Phil. Trans. Roy. Soc., A214, 373-432).

On the convection og heat from small cylinders in a stream of fluid:
Determination of the convection constants of small platinum wires with application to hot-wire anemometry.

$$Nu = (A + B\,\mathrm{Re}^n)(1 + \frac{1}{2} a_T)^m$$

where the dimensionless heat transfer rate (Nusselt number):

$$Nu = \frac{\dot{q}}{\pi l k (T_w - T)}$$
$$= Nu(\mathrm{Re}, \mathrm{Pr}, Gr, M, Kn, a_T, l/d, \theta)$$

The Reynolds number:

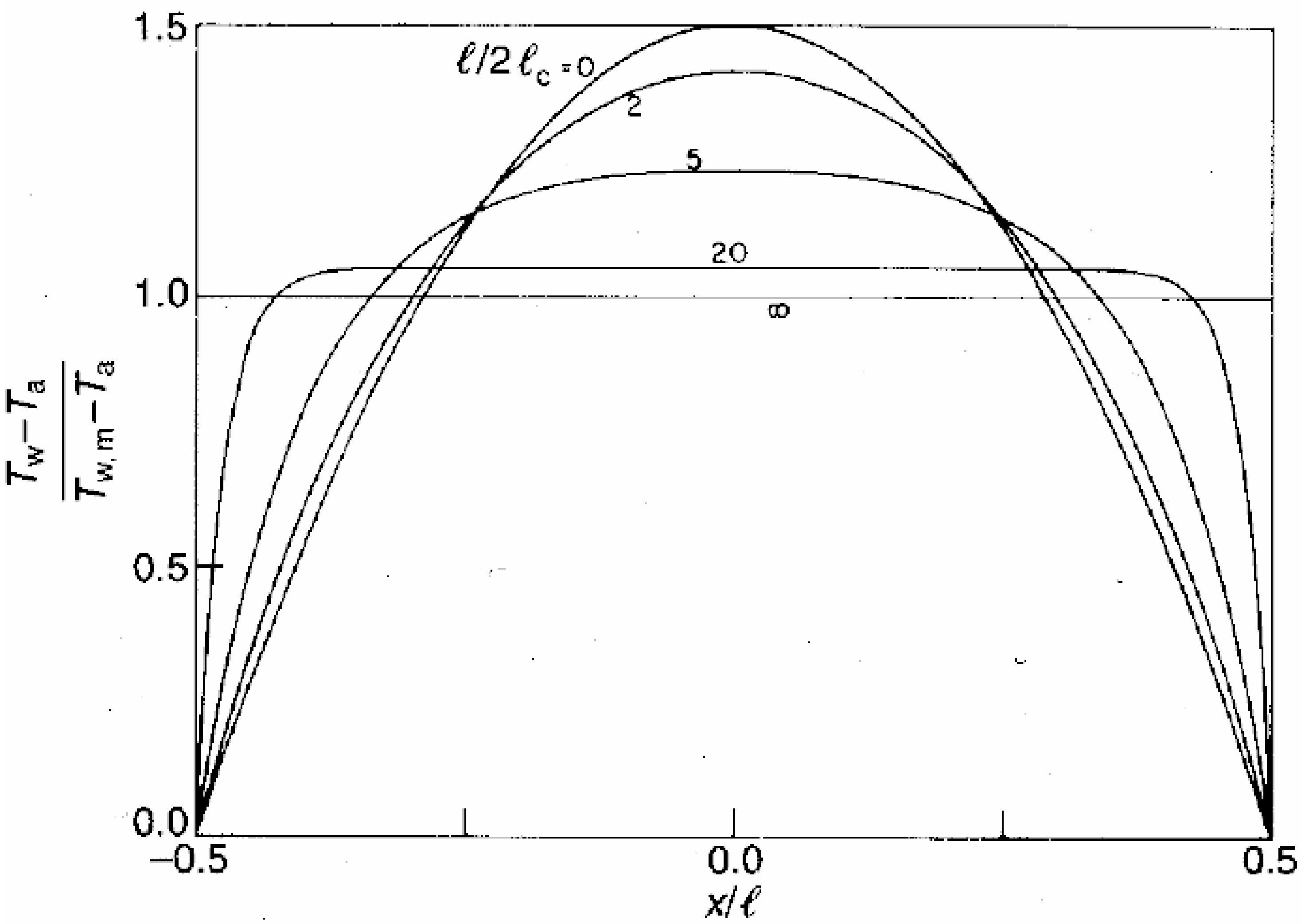
$$\mathrm{Re} = \frac{\rho U d}{\mu}$$

Overheat ratio:

$$a_T = \frac{T_w - T}{T}$$

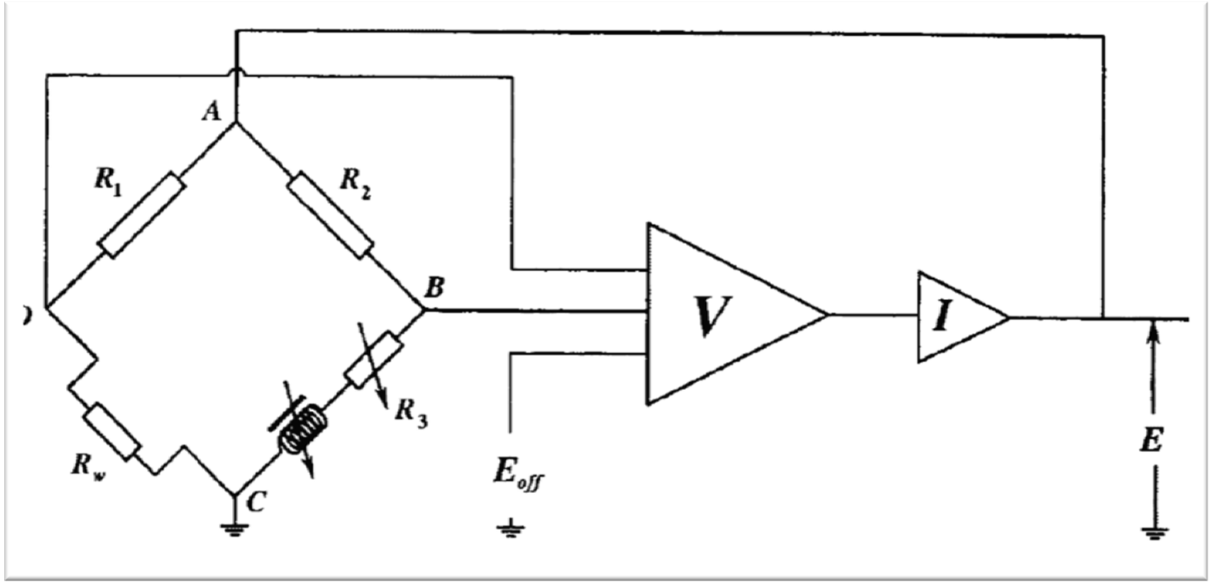
$$\mathbf{I^2 R_w^2 = E^2 = (T_w - T_a)(A + B \cdot U^n)}$$ **“King’s law”**

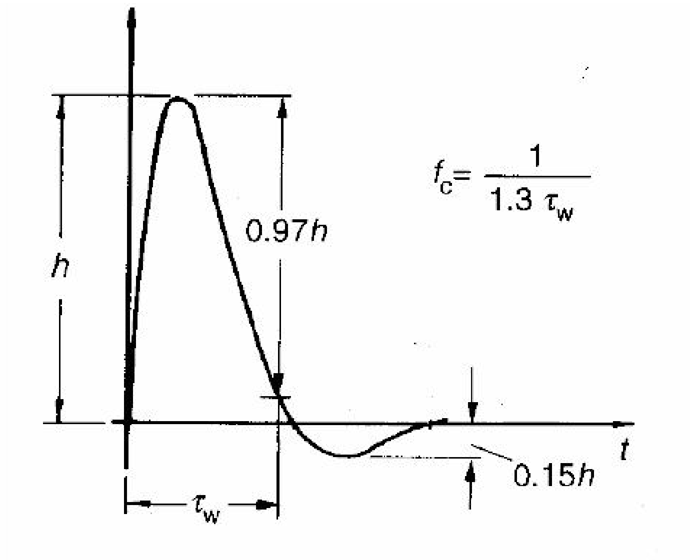
Hot-wire temperature profiles:



Freymuth, 1979

Schematics of the anemometer circuit: Wheatstone bridge





Optimum square-wave test response, **Bruun, 1995**

How it looks in reality:

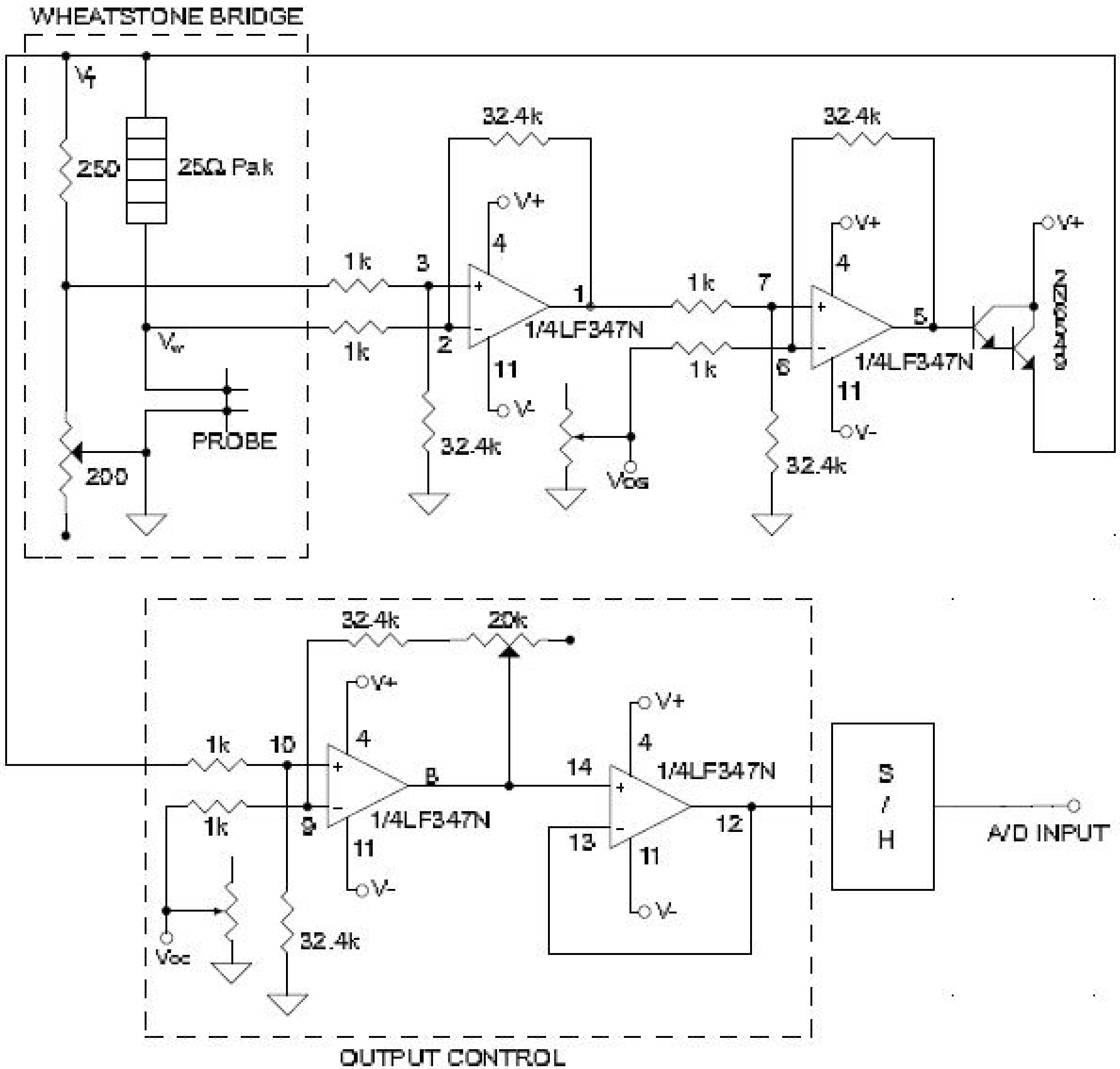


Figure 2.3: Anemometer circuit diagram

Last lecture:

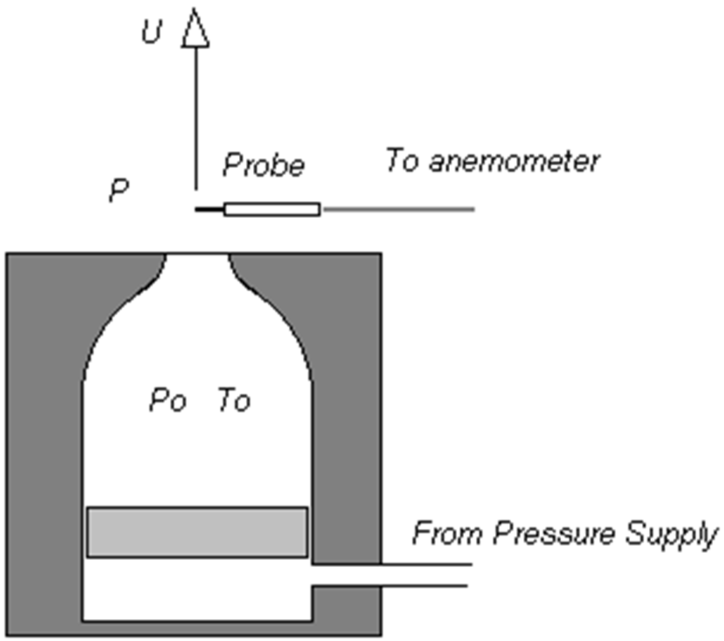
Turbulence
Statistics
Autocorrelation
Integral time scale
Effective number of samples
Record length
Variability of estimator

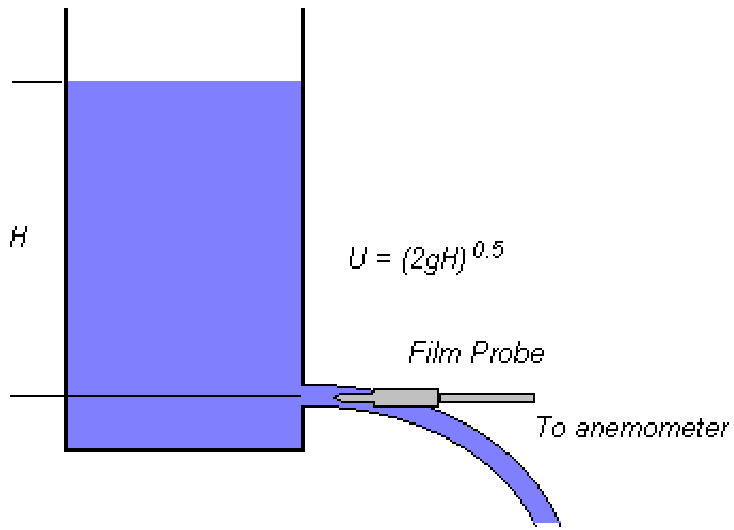
Hot wire anemometry

Today:

Calibration
Some examples
Practical design of experiment.

Calibration before and/or after the experiment is needed in order to convert voltages to velocities.





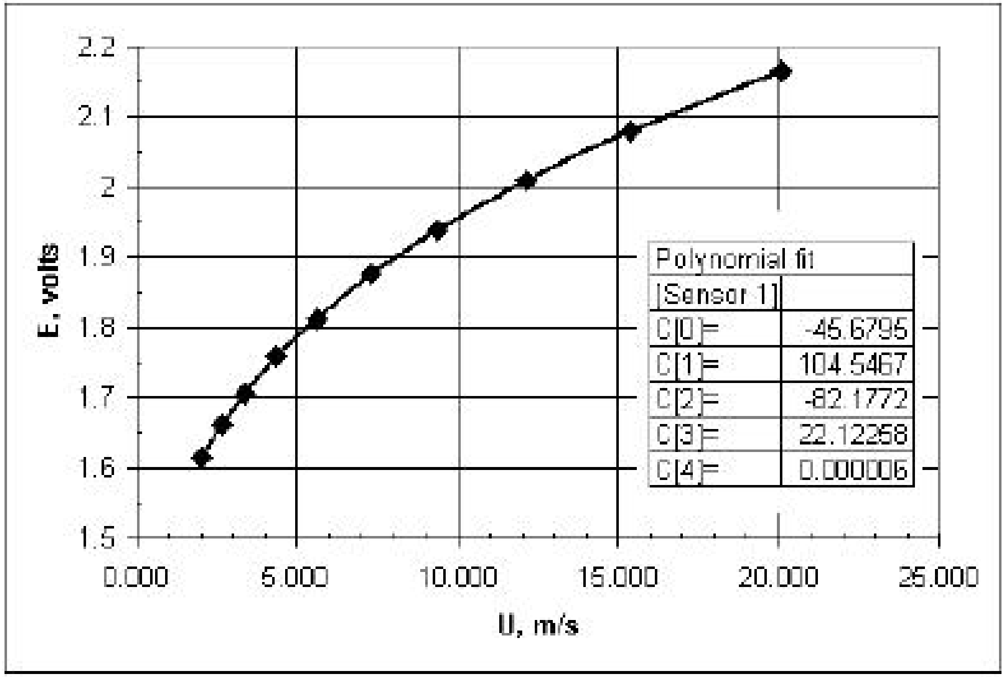
$E^2 = A + Bu^n$ King's Law

$U = C_0 + C_1E + C_2E^2 + C_3E^3 + C_4E^4$ Polynomial calibration

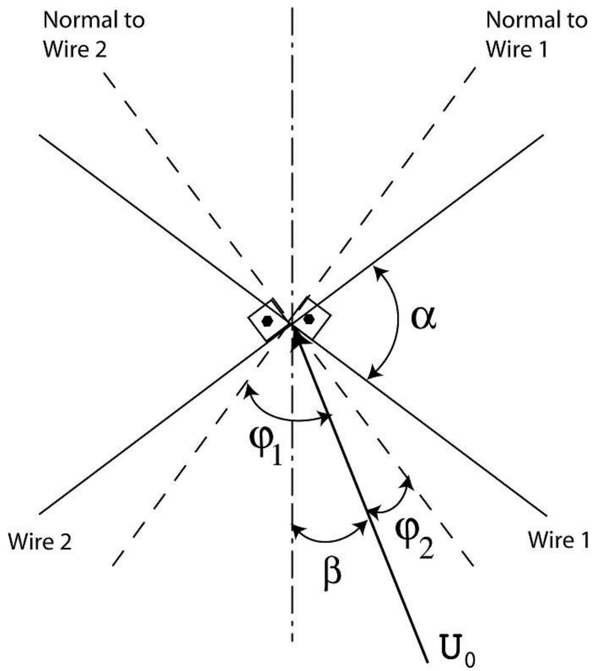
The relation is found by curve fitting to the calibration data using least square method.

Curve fitting of calibration data (manual procedure):

U	E	T	Pbar	Ecorr
m/s	volts	C	Pa	volts
2.019	1.614	26.0	100.652	1.615
2.622	1.661	26.0	100.654	1.662
3.368	1.705	26.0	100.66	1.706
4.360	1.758	25.9	100.663	1.759
5.621	1.813	25.9	100.66	1.814
7.324	1.876	25.9	100.654	1.877
9.379	1.939	25.9	100.652	1.94
12.121	2.01	25.9	100.652	2.011
15.364	2.08	25.9	100.657	2.081
20.101	2.166	25.9	100.657	2.167



Two component measurements need cross-wires; and angular calibration



$$\boxed{U = \sum_{n=0}^{N} c_n E^n}$$

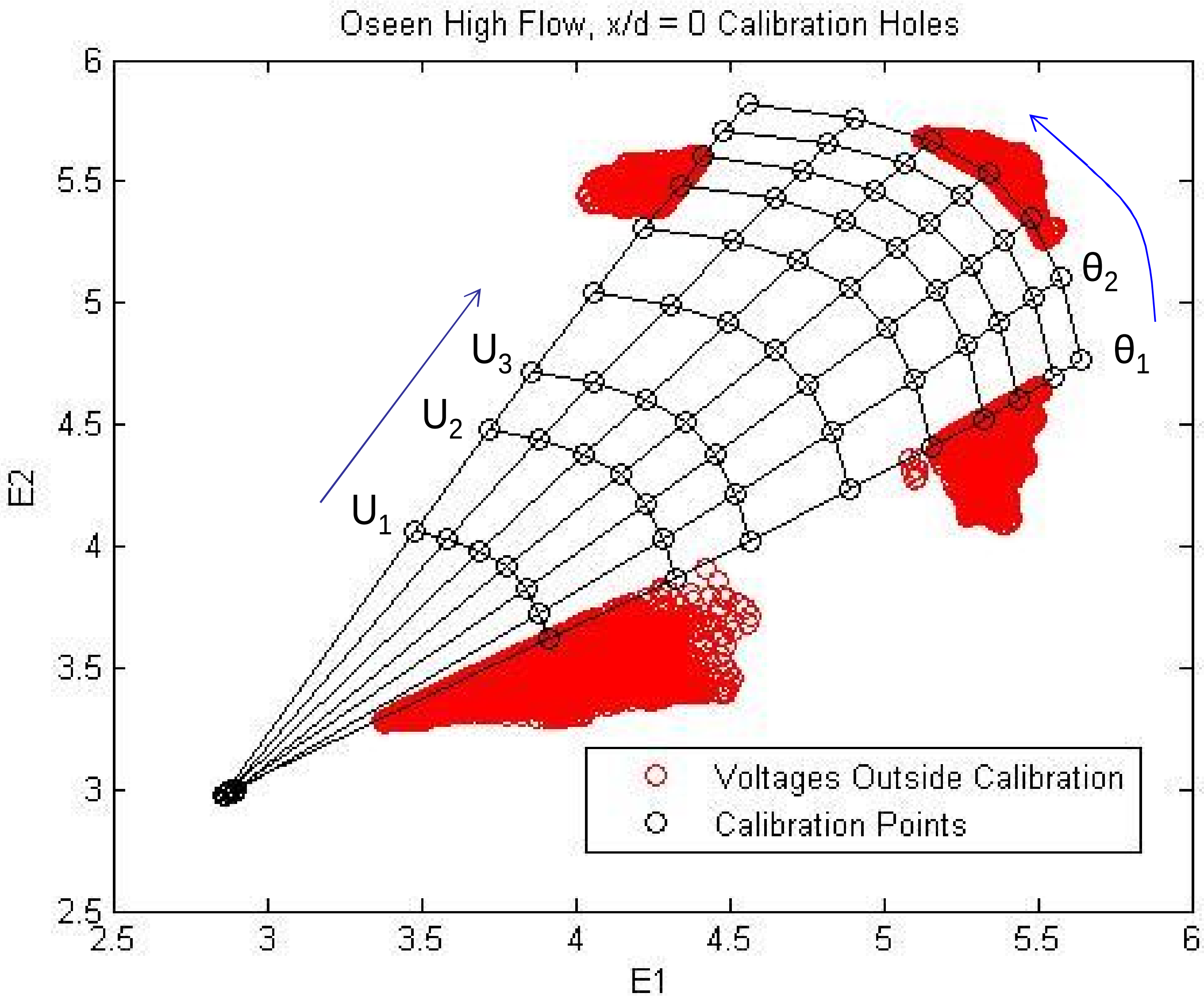
$$U_{eff1} = U_0(cos^2\phi_1 + k_1^2 sin^2\phi_1)^{0.5}$$

$$U_{eff2} = U_0(cos^2\phi_2 + k_2^2 sin^2\phi_2)^{0.5}$$

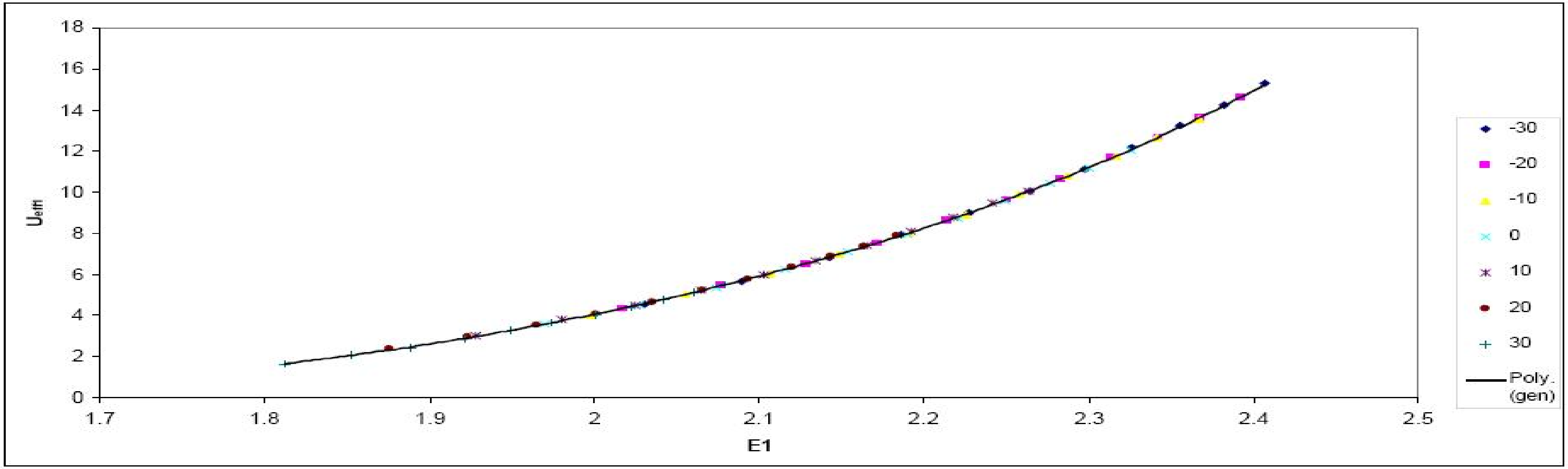
$$\alpha = \phi_1 + \phi_2$$

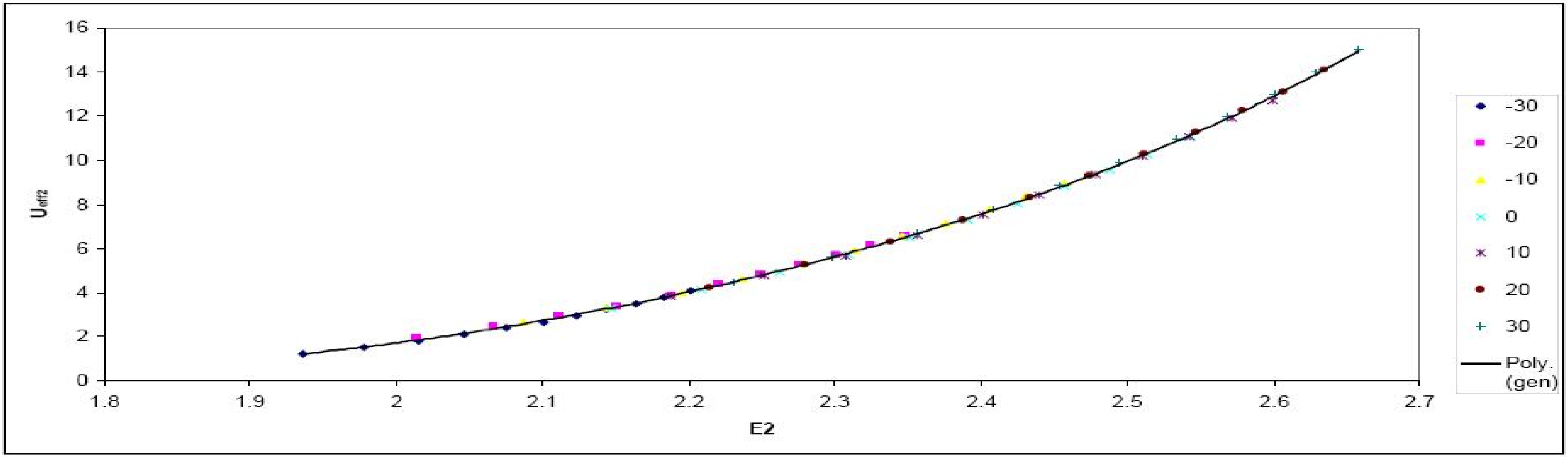
$$\boxed{c_0 + c_1 E + c_2 E^2 + c_3 E^3 + c_4 E^4 - U_0(cos^2\phi + k^2 sin^2\phi)^{0.5} = 0}$$

Calibration of cross-wires may be more troublesome and difficult than expected.



We put all your angular calibrations onto one single curve for each of the sensors!





More difficult and time consuming to perform this way!

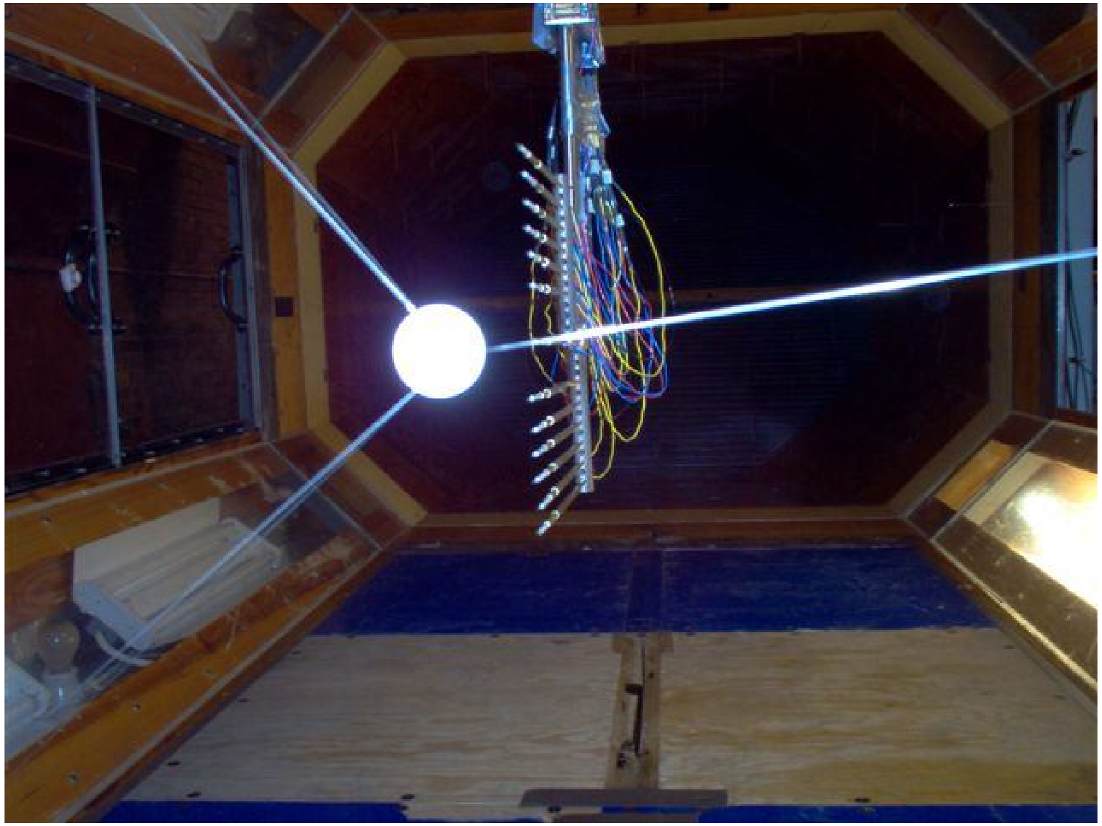
$$(\frac{U_{eff1}}{U_{eff2}})^2 = \frac{cos^2\phi_1 + k_1^2 sin^2\phi_1}{cos^2\phi_2 + k_2^2 sin^2\phi_2}$$

$$\begin{aligned} A &= Fk_2cos^2\alpha + Fsin^2\alpha - 1 \\ B &= 2(F(1-k_2)cos\alpha sin\alpha) \\ C &= Fcos^2\alpha + k_2Fsin^2\alpha - 1 \\ F &= (\frac{U_{eff1}}{U_{eff2}})^2 \end{aligned}$$

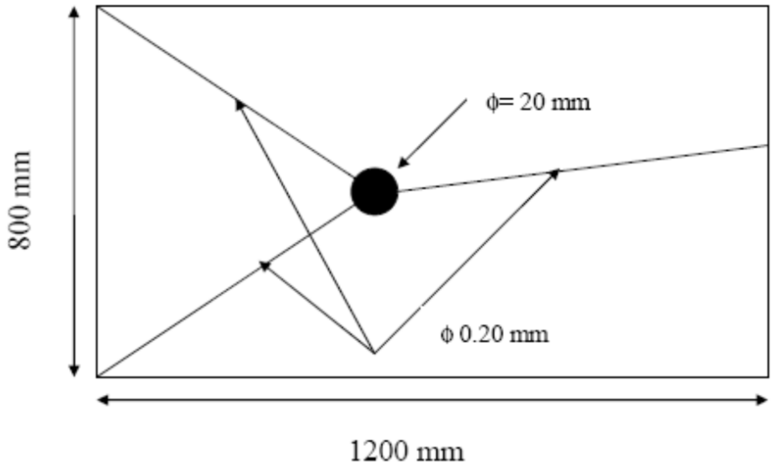
$$\begin{aligned} U &= U_0 cos\beta \\ V &= U_0 sin\beta \end{aligned}$$

$$tan\phi_1 = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

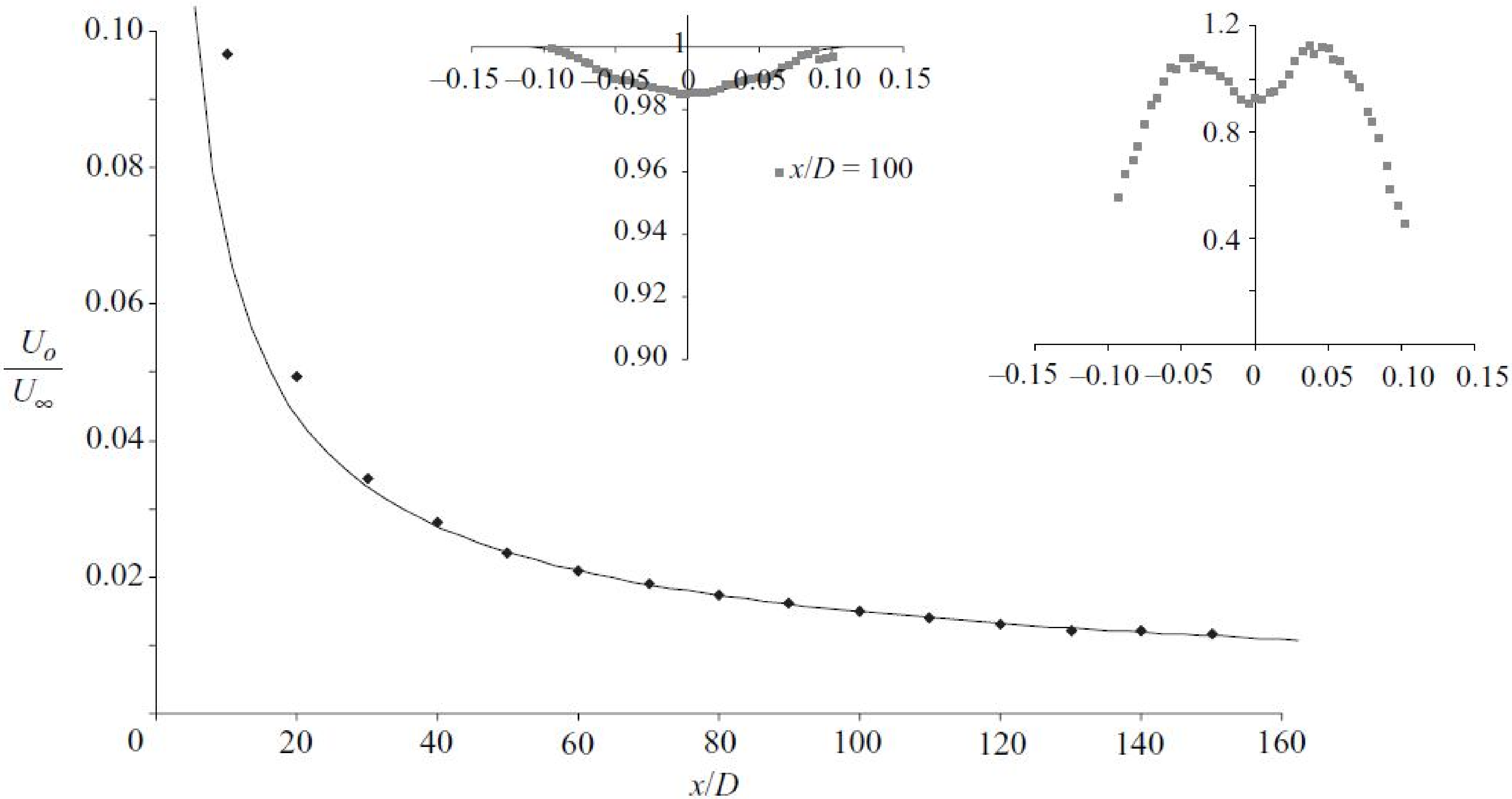
Axisymmetric far wake is very difficult to measure because of small velocity deficit.



U_∞ =15 m/s
D = 20 mm
Re_D = 20000
x/D = 50



Bridge oscillations and quantization error (even for 16 bit A/D converter) are big problems in this case.



Johansson et al, JFM, 2006

21.6 m long wind tunnel of Laboratoire de Mécanique de Lille (LML) is unique to conduct boundary layer research.

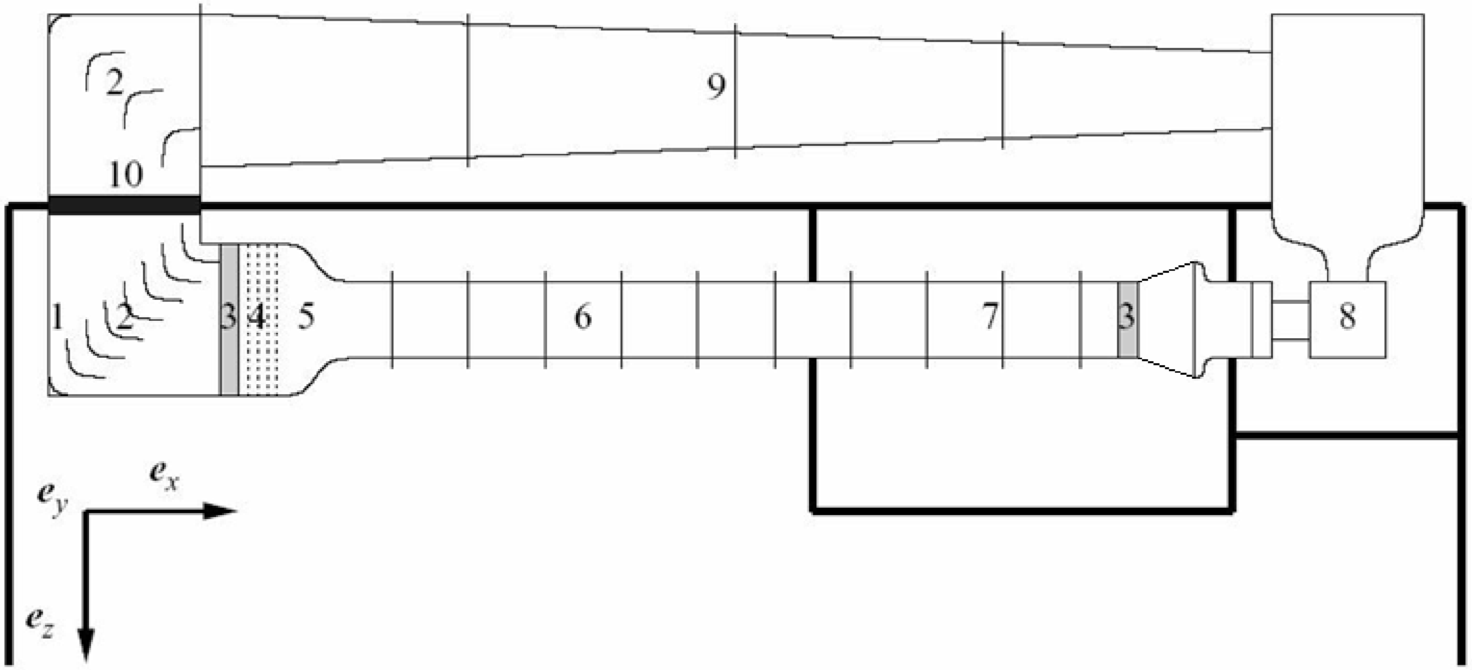
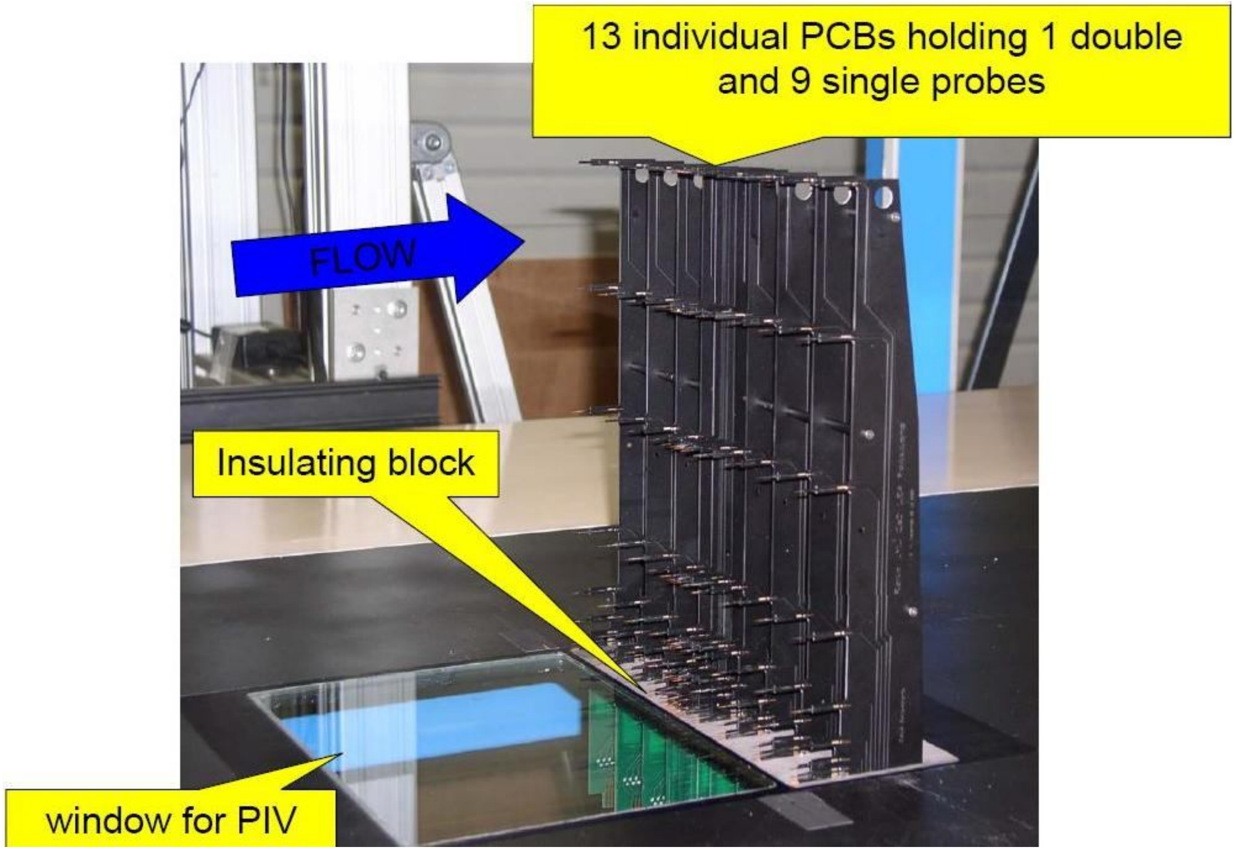


FIGURE 4. Sketch of the top view of the wind tunnel: 1, plenum chamber; 2, guide vanes; 3, honeycomb; 4, grids; 5, contraction; 6, turbulent boundary layer developing zone; 7, testing zone of wind tunnel; 8, fan and motor; 9, return circuit; 10, heat exchanger (air–water).

U_∞ (m/s)	u_τ (m/s)	δ (m)	$y^+=1$ (µm)	Re_θ
5	0.185	0.323	80	11 500
10	0.350	0.302	40	20 600

A hot-wire rake of 143 single wire probes to get both spatial and temporal information about the flow.



Probe positioning is crucial

z-position : 0, ± 4.0 mm, ± 12.0 mm, ± 28.0 mm, ± 60.0 mm, ± 100.0 mm, ± 145.0 mm

y-position : 0.3 mm, 0.9 mm, 2.1 mm, 4.5 mm, 9.3 mm, 18.9 mm, 38. 1 mm, 76.5 mm, 153.3 mm, 230.1 mm, 306.9 mm

Some practical info: Fast (spectral) or slow measurement

Example: pipe flow

Large scale ~ R
Characteristic velocity ~ $U_{centerline}$
Time scale of large scales ~ R/U_{cl}

Small scale: Kolmogorov microscale

$$\eta_k = \left(\frac{\nu^3}{\epsilon}\right)^{1/4} \qquad \epsilon \sim \frac{u^3}{\ell}$$

$$\tau_k = \left(\frac{\nu}{\epsilon}\right)^{1/2}$$

$$l_{probe} < \lambda/2$$

$$k_d = \frac{1}{\eta_k} \quad \Rightarrow \quad \lambda_d = \frac{2\pi}{k_d} = 2\pi\eta_k$$

$$l_{probe} = \pi\eta_k$$

$$k_1 = \frac{2\pi f}{U}$$

Cut-off frequency in practice : $f_c = \dfrac{U_c}{2 \times \ell_w}$

$$\varepsilon_{e^n}^2 = \frac{2I}{T}\left(\frac{\mathrm{var}(\mathrm{e^n})}{\langle e^n\rangle^2}\right) = \frac{2I}{T}\left(\frac{\langle e^{2n}\rangle - \langle e^n\rangle^2}{\langle e^n\rangle^2}\right)$$

$$\langle e^n\rangle = \frac{n!\sigma^n}{2^{n/2}(n/2)!}$$

$$\langle e^2\rangle = \sigma^2,\ \langle e^4\rangle = 3\sigma^4,\ \langle e^6\rangle = 15\sigma^6$$