MEK4600: WaveLab 1

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Analysis of time series

For this WaveLab you will be working with time series of surface elevation $\eta(x,t)$ which you have collected from the large wavetank in the Hydrodynamic laboratory. The data is collected with a stationary array consisting of sixteen ultrasonic probes. In the stationary array the ultrasonic probes are equispaced with distance $\Delta x = 0.3$ m and for all probes the data is collected at a sampling rate of $s = 200$ Hz. The technical specifications of the ultrasonic probes can be found online at ... The array is mounted on rails along the length of the wavetank and this allows us to perform dense spatio-temporal synthetic measurements of the wave fields.

You will measure two irregular wave fields with steepness:

$$\epsilon = a_c k_p, \tag{1}$$

equal to 0.02 and 0.10 initially closest to the wave generator, where $a_c = \sqrt{2\langle\eta^2\rangle}$ is the characteristic amplitude of the irregular wave field, $\langle\eta^2\rangle$ is the mean-square of the time series of the surface elevation measured by the probe closest to the wave generator and k_p is the peak wavenumber. The goal of the WaveLab is to compare the directly measured dispersion relation with the linear dispersion relation for water surface gravity waves:

$$\omega^2 = gk, \tag{2}$$

and possible discover if there is any qualitative difference between linear and nonlinear wave regimes when the initial steepness ϵ increases from 0.02 to 0.10.

For each synthetic measurement the data will be collected in lvm files which consists of seventeen columns. The first coloumn is the discrete time steps and the second to seventeenth column is the surface elevation data from the sixteen probes in the stationary array. The second column is the measurement of the surface elevation from the probe closest to the wave generator and the seventeenth coloumn is the measurement from the probe closest to the damping beach.

1. Store the lvm data you collected in the laboratory from the wave field with steepness ϵ equal to 0.02 in a folder on a computer and plot the time series of the raw data from the measurement closest to the wave generator. This can be done by using the matlab commands:

```
>> load e0,02_Run01_d00.lvm
>> A=e0,02_Run01_d00
>> plot(A(:,1),A(:,2))
>> axis([0 120 -3 3])
```

Do a visual inspection of the time series (the axis command is useful in this context). Are there any anomalies? If any, how can you describe them? Suggested reading section 10.4.2 in Bendat and Piersol (2000).

2. Store the lvm data from the wave field with steepness ϵ equal to 0.10 in another folder. Do a visual inspection of the raw data time series. Does the number of anomalies increase compared to the wave field with ϵ equal to 0.02? If so, what do you think is a reasonable explanation?

   Download the matlab m-files e02SignalProcessing.m and e10SignalProcessing.m from the online link given above and store the m-files in the same folders as the data collected from the laboratory. Run e02SignalProcessing.m and e10SignalProcessing.m. The outputs are $M \times N$-matrixes ETA were M is the number of synthetic measurement positions in space and N is the number of time samples taken at each probe. The columns in ETA are ordered so that the first column corresponds to the measurement closest to the wave generator and the last column corresponds to the measurement closest to the damping beach. The synthetic measurements are equispaced with $\Delta x = 0.05$ m.

3. Plot the time series closest to the wave generator for both wave fields (you are free to write the plot command in to the already existing m-file). This can be done by for instance:

```
plot(t,ETA(:,1))
```

   Do a visual inspection of the processed time series. Are there now any differences compared to the raw data plotted in exercise 1 and 2? What do you think the downloaded processing m-files do? Do you see any noise in the data? You can filter the data by using a smoothing filter, for instance a Savitzky-Golay filter implemented in Matlab®:

```
po=3;     % Polynomial order
ffs=61;   % Filter frame size
ETA=sgolayfilt(ETA,po,ffs);
```

   Feel free to experiment with different paramenters for the filter. What happens with the surface elevation if you use large polynomial orders and filter frame sizes?

4. Compute the discrete angular frequency spectrum $S(\omega_n)$ from the measured time series closest to the wave generator for both wave fields. This can be done in the following manner. Discretize the wave angular frequencies in steps of:

$$\Delta\omega = \frac{2\pi}{T} \tag{3}$$

   where T is the total time length of the time series and $T = N\Delta t$ with $\Delta t = 1/s$. The angular frequencies $\omega_n = n\Delta\omega$ where $n = 0, 1, 2, ..., N-1$. The spectral energy density $S(\omega_n) = |\hat{\eta}(\omega_n)|^2$ were $\hat{\eta}(\omega_n)$ is the one-dimensional Fourier transform of the time series $\eta(t_n)$. In matlab® the discretization of the angular frequencies is easily achived with the command:

```
n=0:N-1;
do=(2*pi)/T;
o = n*do;
```

and the one-dimensional Fourier-transform is computed from the FFT-algorithm:

```
etaHat=ifft(ETA(:,1))
```

The spectral energy density:

```
So=abs(etaHat)^2
```

Plot the angular frequency spectrum $S(\omega_n)$ by using the command:

```
plot(o,So)
axis([0 20 0 1.0*10^(-3)])
```

Adjust the axis if necessary. Can you describe the spectrum? Hint: the curve may not look very nice.

5. Make the curve nicer by smoothing the spectral energy density:

```
po=1;
ffs=31;
So=sgolayfilt(So,po,ffs);
```

Plot the smoothed angular frequency spectrum. Is there any difference compared to when the smoothing is not used? Find the maximum of the spectrum and the corresponding angular frequency. We will name this angular frequency the peak angular frequency ω_p. Find the peak wavenumber $k_p = \omega_p^2/g$ from the linear dispersion relation in Eq. (2) and the steepness ϵ from Eq. (1) for both wave fields. To compute the mean of the time series use:

```
mean(ETA(:,1))
```

Verify that the initial steepness of the wave fields are ϵ equal to 0.02 and 0.10. The angular frequency has unit rads^{-1} and the wavenumber has unit radm^{-1}.

Compare the measured data with the linear dispersion relation for water surface gravity waves $\omega^2 = gk$. We will do this in the following manner. Compute a two-dimensional Fourier-transform of the space-time series collected in ETA and compute the spectral energy density $S(k_m, \omega_n)$. This can be done from the commands:

```
ETAHat=(1/M)*(fft(ifft(ETA).').');
S=abs(ETAHat)^2;
```

The wavenumbers are discretized in steps of,

$$\Delta k = \frac{2\pi}{M\Delta x} \tag{4}$$

where $M\Delta x = L$ is the length of the synthetic array in the wavetank. The wavenumbers $k_m = m\Delta k$ where $m = 0, 1, 2, ..., M-1$.

6. Plot the spectral energy density in dB (decibel). This is done by taking the logarithm of the normalized spectral energy density and multiplying it with ten:

```
maxS= max(S(:));
SdB=10*log10(S/maxS);
```

Make the wavenumber and angular frequency axis non-dimensional. This is done simply by dividing the discretized wavenumber and angular frequencies with the peak wavenumber k_p and the peak angular frequency ω_p respectively. For instance:

```
k_p=?
o_p=?
k=k./k_p;
o=o./o_p;
```

We will make a k/ω-spectrum by plotting the contours of the spectral energy density in k/ω-space. For the k/ω-spectrum it is recommended not to use a smoothing filter on the surface elevation. To plot the k/ω-spectrum use the commands:

```
pcolor(k,o,SdB)
shading interp
axis([0 5 0 5])
caxis([-70 0])
```

Plot the linear dispersion relation $\omega = \sqrt{gk}$ in the same figure. If you like you can use a finer resolution for the linear dispersion relation curve. Adjust the axis if necessary. The highest energies will be shown in red colours. How is the spectral energy density distributed compared to the linear dispersion relation? Is there any qualitative difference between the linear (ϵ equal to 0.02) and nonlinear (ϵ equal to 0.10) wave regimes?

We will also do a simple error analysis on the repetition error between three independent repetitions of the wave field which were measured from the array position d00 closest to the wave generator.

7. Download the m-file errorSignalProcessing.m. Let $R = 1, 2, 3$ denote the wave runs Run01, Run02 and Run03 and let the surface elevation measured closest to the wave generator from these three runs be $\eta_R(t_n)$. Calculate the repetition errors (in percentage) from the independent measurements closest to the wave generator by,

$$\zeta_{1,2} = \frac{\sum_{n=0}^{N-1}(\eta_1(t_n) - \eta_2(t_n))^2}{\sum_{n=0}^{N-1}(\eta_1(t_n))^2} \tag{5}$$

and

$$\zeta_{1,3} = \frac{\sum_{n=0}^{N-1} (\eta_1(t_n) - \eta_3(t_n))^2}{\sum_{n=0}^{N-1} (\eta_1(t_n))^2} \tag{6}$$

How are the repetition errors? Would you say that the wave field is repeatable? How significant are the repetition errors for using the synthetic array?