Time Value of Money.

1) NOK 90 000 $\xrightarrow[\text{simple interest rate } r]{\text{Invested for 61 days}}$ NOK 90 200

Find the interest rate r and the net return on the investment

The rule of simple interest is $V(t) = (1 + rt)V(0)$ $t \geq 0$

In this case we have $t = \frac{61}{365}$, $V(0) = 90.000$

and $V\left(\frac{61}{365}\right) = 90\,200$, which yields

$$r = \frac{365}{61}\left(\frac{90,200}{90,000} - 1\right) \simeq 0'0133 = 1'33\%$$

The rate of return on the investment is given by the formula

$$R\left(0, \frac{61}{365}\right) = \frac{V\left(\frac{61}{365}\right) - V(0)}{V(0)} = \frac{200}{90,000} = \frac{2}{900} \approx 0'0622 = 6'22\%$$

$\boxed{2}$ $\quad NOK\ 80\,000 \xrightarrow[\text{Simple interest rate } 9\%]{\text{Invested } t \text{ days}} NOK\ 83\,000$

Find t and the rate of return of the investment.

The rule of simple interest is $V(t) = (1 + rt)V(0) \quad t \geq 0.$

In this case $\quad V(t) = 83{,}000$

$V(0) = 80{,}000$

$r = 9\% = 0'09$

This yields

$$t = \frac{1}{r}\left(\frac{V(t)}{V(0)} - 1\right) = \frac{1}{0'09}\left(\frac{83{,}000}{80{,}000} - 1\right) \cong 0'4167$$

$$\simeq \frac{152'096}{365} \Rightarrow t \simeq 152'096 \text{ days.}$$

The rate of return will be

$$R(0,t) = \frac{V(t) - V(0)}{V(0)} = \frac{83{,}000 - 80{,}000}{80{,}000} = \frac{3}{80} = 0'0375$$

$$= 3'75\%$$

$\boxed{4}$ $P \xrightarrow[\text{Compounded daily with rate } 6\%]{\text{Invested for } t \text{ years}} 2P$

Find t

The future value of an investment growing with periodic compounding (m periods a year) with rate r is given by

$$V(t) = P\left(1 + \frac{r}{m}\right)^{tm} \quad t \geq 0$$

In this case $V(t) = 2P$, $r = 0{,}06$, $m = 365$, which yields

$$2P = P\left(1 + \frac{0{,}06}{365}\right)^{365t} \iff 2 = \left(1 + \frac{0{,}06}{365}\right)^{365t}$$

Taking logarithms we get

$$\log(2) = 365\,t\,\log\left(1 + \frac{0'05}{365}\right)$$

$$\Rightarrow t = \frac{\log(2)}{365\log\left(1+\frac{0'05}{365}\right)} \simeq 11'5534 \text{ years}$$

That is 11 years and $202'991 \simeq 202$ days

$\boxed{6}$ $V(0) \xrightarrow{\text{Invested for 100 years}} 10^5$ NOK

Compounded a) daily with rate 5%

b) annually

Find $V(0)$ in the case a) and b).

The present value of a future amount discounted with periodic compounding (m periods a year) with rate r is given by

$$V(0) = V(t)\left(1 + \frac{r}{m}\right)^{-tm}$$

We have $t = 100$, $V(100) = 10^5$ and $r = 0'05$

a) $m = 365$, which gives

$$V(0) = 10^5 \left(1 + \frac{0{,}05}{365}\right)^{-100 \cdot 365} = 674'025 \text{ NOK}$$

b) $m = 1$, which gives

$$V(0) = 10^5 \left(1 + \frac{0{,}05}{1}\right)^{-100 \cdot 1} = 760'449 \text{ NOK}$$

7) Which is greater, the interest rate r or the rate of return $R(0,1)$ if the compounding frequency m is greater than one?

The rate of return of an a deposit attracting interest compounded periodically is given by

$$R(s,t) = \frac{V(t) - V(s)}{V(s)} = \left(1 + \frac{r}{m}\right)^{(t-s)m} - 1$$

Hence, we have to check whether

$$R(0,1) = \left(1 + \frac{r}{m}\right)^{m} - 1 > r \qquad (*)$$

or the reversed inequality holds, whenever $\boxed{m > 1}$

The idea is to use the Binomial formula

$$(x+y)^n = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} x^k y^{n-k}$$

with $x = 1$, $y = r/m$ and $n = m$

For $m \geq 2$, $m \in \mathbb{N}$, we get

$$\left(1 + \frac{r}{m}\right)^m = 1 + r + \underbrace{\sum_{k=2}^{m} \frac{m!}{k!(m-k)!} \left(\frac{r}{m}\right)^k}_{> 0} \qquad (**)$$

This clearly yields that (combining (*) with (**))

$$R(0,1) = r + \underbrace{\sum_{k=2}^{m} \frac{m!}{k!(m-k)!} \left(\frac{r}{m}\right)^k}_{> 0} \Rightarrow R(0,1) > r.$$