$\boxed{8}$ Amortized loan of 10^6 NOK with 5 yearly installments and interest of 15% annually.
Analyze in detail the loan.

In general, assume N years, interest rate r and the amount P.
Then, the installments are given by

$$C = \frac{P}{PA(N,r)} = \frac{P}{\frac{1-(1+r)^{-N}}{r}} = \frac{rP}{1-(1+r)^{-N}}$$

Let $n = 1, \dots, N$. The present value of the outstanding balance after $n-1$ installments is equal to the amount borrowed P reduced by the present value of the first $n-1$ installments

$$P - C \times PA(n-1, r) = P - \frac{rP}{1-(1+r)^{-N}} \cdot \frac{1-(1+r)^{-(n-1)}}{r}$$

$$= P\left(1 - \frac{1-(1+r)^{-(n-1)}}{1-(1+r)^{-N}}\right)$$

$$= P\left(\frac{\cancel{1} - (1+r)^{-N} + \cancel{1} + (1+r)^{-(n-1)}}{1-(1+r)^{-N}}\right)$$

$$= P\left(\frac{(1+r)^{N-(n-1)} - 1}{(1+r)^{N} - 1}\right)$$

The actual outstanding balance remaining after $n-1$ instalments, can be found by computing its future value, which gives

$$(1) \quad P\left(\frac{(1+r)^{N-(n-1)} - 1}{(1+r)^{N} - 1}\right)(1+r)^{n-1} = P\left(\frac{(1+r)^{N} - (1+r)^{n-1}}{(1+r)^{N} - 1}\right)$$

The interest included in the nth instalment is the previous amount multiplied by r.

$$(2) \quad P\left(\frac{(1+r)^N-(1+r)^{n-1}}{(1+r)^N-1}\right) r$$

The capital repaid as part of the nth instalment is the difference between the outstanding balance after the $(n-1)$th instalment and the nth instalment

$$P\left(\frac{(1+r)^N-(1+r)^{n-1}}{(1+r)^N-1}\right) - P\left(\frac{(1+r)^N-(1+r)^{n}}{(1+r)^N-1}\right) = P\left(\frac{(1+r)^n-(1+r)^{n-1}}{(1+r)^N-1}\right)$$

$$= P\,\frac{(1+r)(1+r)^{n-1}-(1+r)^{n-1}}{(1+r)^N-1} = P\,\frac{r(1+r)^{n-1}}{(1+r)^N-1} \qquad (3)$$

Setting $P = 10^6$, $r = 0'15$

t (years)	interest paid (2)	Capital repaid (3)	Outstanding balance (4)
0	—	—	10^6
1	150,000	148,316	851,684
2	127,753	170,563	681,121
3	102,168	196,147	484,974
4	72,747	225,569	259,405
5	38,911	259,405	0

$C = 298316$

9. How much can you borrow?

- 18 % interest rate
- You can pay 10^5 NOK each year
- Clear the loan in 10 years

This is an amortised loan and can be seen as an annuity. The present value of an annuity with payments of C for n years and rate r is given by

$$V(0) = C \times PA(n, r)$$

where $PA(n, r) = \frac{1 - (1+r)^{-n}}{r}$ is the present value factor.

In this case $C = 10^5$, $n = 10$, $r = 0'18$ which yields that you can borrow

$$V(0) = 10^5 \cdot \frac{1 - (1'18)^{-10}}{0'18} = 449{,}408$$

20

- Deposit NOK 12000 at the end of each year
- For 40 years
- Annual compounding of 8%

Find the balance after 40 years.

The present value of the balance can be found using the formula for an annuity, which is given by

$$V(0) = C \times PA(r, n) = C \, \frac{1-(1+r)^{-n}}{r}$$

Then we can use the future value formula with annual compounding

$$V(t) = (1+r)^t \, V(0) \qquad t \geq 0.$$

to find the balance.

Here we have $C = 12000$ $r = 0'05$ and $n = 40$

This gives

$$V(0) = 12,000 \, \frac{1-(1+0'05)^{-40}}{0'05} = 205,909$$

and

$$V(40) = 205,909 \, (1+0'05)^{40} \simeq 1'449 \times 10^6$$