$\boxed{6}$

b) Exercise 2 a) we know that $G^* = (H_1, -H_1, -2H_1)^T$

$$W = \{ X = (X_1, X_2, X_3)^T \in \mathbb{R}^3 : X = G^* \}$$

$$= \{ X \in \mathbb{R}^3 : X_1 = H_1,\ X_2 = -H_1,\ X_3 = -2H_1, \text{ for some } H_1 \in \mathbb{R} \}$$

$$= \{ X \in \mathbb{R}^3 : X_1 + X_2 = 0,\ 2X_1 + X_3 = 0 \}$$

Note that $\dim(W) = 1$ and a basis for W is $\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$

$W^\perp$ consists of all $Y \in \mathbb{R}^3$ that are orthogonal to $\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$

$$(1 \ \ -1 \ \ -2) \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = Y_1 - Y_2 - 2Y_3 = 0. \text{ Therefore}$$

$$W^\perp = \{ Y = (Y_1, Y_2, Y_3)^T \in \mathbb{R}^3 : Y_1 = Y_2 + 2Y_3 \}$$

$$\begin{pmatrix} Y_2 + 2Y_3 \\ Y_2 \\ Y_3 \end{pmatrix} = Y_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + Y_3 \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

basis of $W^\perp$