An Introduction to Mathematical Finance
UiO-STK-MAT3700
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Exercises 2

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Basic Financial Derivatives

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1. Suppose that $S(0) = 170$NOKs, $F(0, 1) = 180$NOKs, $r = 8$, and short-selling requires a 30% security deposit attracting interest at $d = 4\%$. Is there an arbitrage opportunity? Find the highest rate d for which there is no arbitrage opportunity.

2. Suppose that the price of stock on 1 April 2018 turns out to be 10% lower than it was on 1 January 2018. Assuming that the risk-free rate is constant at $r = 6\%$, what is the percentage drop of the forward price on 1 April 2018 as compared to that on 1 January 2018 for a forward contract with delivery on 1 October 2018?

3. Let r be the risk-free rate. a) Prove that $F(t, T)$, the forward price for a stock paying a known dividend D at a future time $0 \leq t_D \leq T$, is given by

$$F(t, T) = \left(S(t) - De^{-r(t_D - t)}\right) e^{r(T-t)} \qquad 0 \leq t \leq t_D.$$

   b) Prove that if the divident is being paid continuously over the period $[0, T]$ at a rate r_D, then the formula is given by

$$F(t, T) = S(t)\, e^{(r - r_D)(T-t)}, \qquad 0 \leq t \leq T.$$

4. Let $V(t)$ be the value at time $t \leq T$ of a forward contract with forward price $F(0, T)$ and delivery time T. Show that $V(t) > (F(t, T) - F(0, T))\, e^{-r(T-t)}$ leads to an arbitrage opportunity.

5. Suppose that the price of a stock is NOK450 at the beginning of the year, the risk-free rate is 6%, and a NOK20 dividend is to be paid after half a year. For a long forward position with delivery in one year, find its value after 9 months if the stock price at that time turns out to be a) NOK490, b) NOK450.

6. Suppose the interest rate r is constant. Given $S(0)$, find the price of the stock after one day such that the marking to market of futures with delivery in 3 months is zero on that day.

7. Find the stock price on the exercise date for a European put option with strike price NOK360 and exercise date in three months to produce a profit of NOK30 if the option is bought for NOK45, financed by a loan at 12% compounded continuously.

8. Find the expected gain (or loss) for a holder of a European call option with strike price NOK90 to be exercised in 6 months if the stock price on the exercise date may turn out to be NOK87, NOK92 or NOK97 with probability 1/3 each, given that the option is bought for NOK8, financed by a loan at 9% compounded continuously.

9. Suppose that a stock paying no dividends is trading at NOK156 a share. European calls on the stock with strike price NOK150 and exercise date in three months are trading at NOK28.3. The interest rate is $r = 6.72\%$, compounded continuously. What is the price of a European put with the same strike price and exercise date?

10. European call and put options with strike price NOK240 and exercise date in six months are trading at NOK50.9 and NOK77.8. The price of the underlying stock is NOK203.7 and the interest rate is 7.48%. Find an arbitrage opportunity.

11. Prove by a rigorous arbitrage argument that

    (a) $0 \leq C^E \leq C^A$,

    (b) $0 \leq P^E \leq P^A$.

12. Recall the non-negative part function $(x)^+ = \max(x, 0)$. Let $0 \leq K_1 < K_2$, $\alpha \in (0, 1)$ and $K = \alpha K_1 + (1 - \alpha) K_2$. Prove that

$$(x - K)^+ \leq \alpha (x - K_1)^+ + (1 - \alpha)(x - K_2)^+ .$$

13. Prove that, on a stock paying no dividends, one has that

    (a) $\left(S(0) - Ke^{-rT}\right)^+ \leq C^A < S(0)$.

    (b) $(K - S(0))^+ \leq P^A < K$.

14. A European call option with a strike price of NOK50 costs NOK2. A European put option with a strike price of NOK45 costs NOK3. Explain how a strangle can be created from these two options. What is the pattern of profits from the strangle?

15. What trading position is created from a long strangle and a short straddle when both have the same time to maturity? Assume that the strike price in the straddle is halfway between the two strike prices of the strangle.